

Computer Graphics

- Texturing -

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Texture

- **Textures modify the input for shading computations**
 - Either via (painted) images textures or procedural functions
- **Example texture maps for**
 - Reflectance, normals, shadows, reflections, essentially anything, ...



Definition: Textures

- **Textures map texture coordinates to shading values**
 - Input: 1D/2D/3D/4D texture coordinates
 - Explicitly given or derived via other data (e.g., position, direction, ...)
 - Output: Scalar or vector value
 - **Modified values in shading computations**
 - Reflectance
 - Changes the diffuse or specular reflection coefficient (k_d, k_s)
 - Geometry and Normal (important for lighting)
 - Displacement mapping $P' = P + \Delta P$
 - Normal mapping $N' = N + \Delta N$
 - Bump mapping $N' = N(P + tN)$
 - Opacity
 - Modulating transparency (e.g., for fences in games)
 - Illumination
 - Light maps, environment mapping, reflection mapping
 - Anything else ...
-

IMAGE TEXTURES

Image Textures

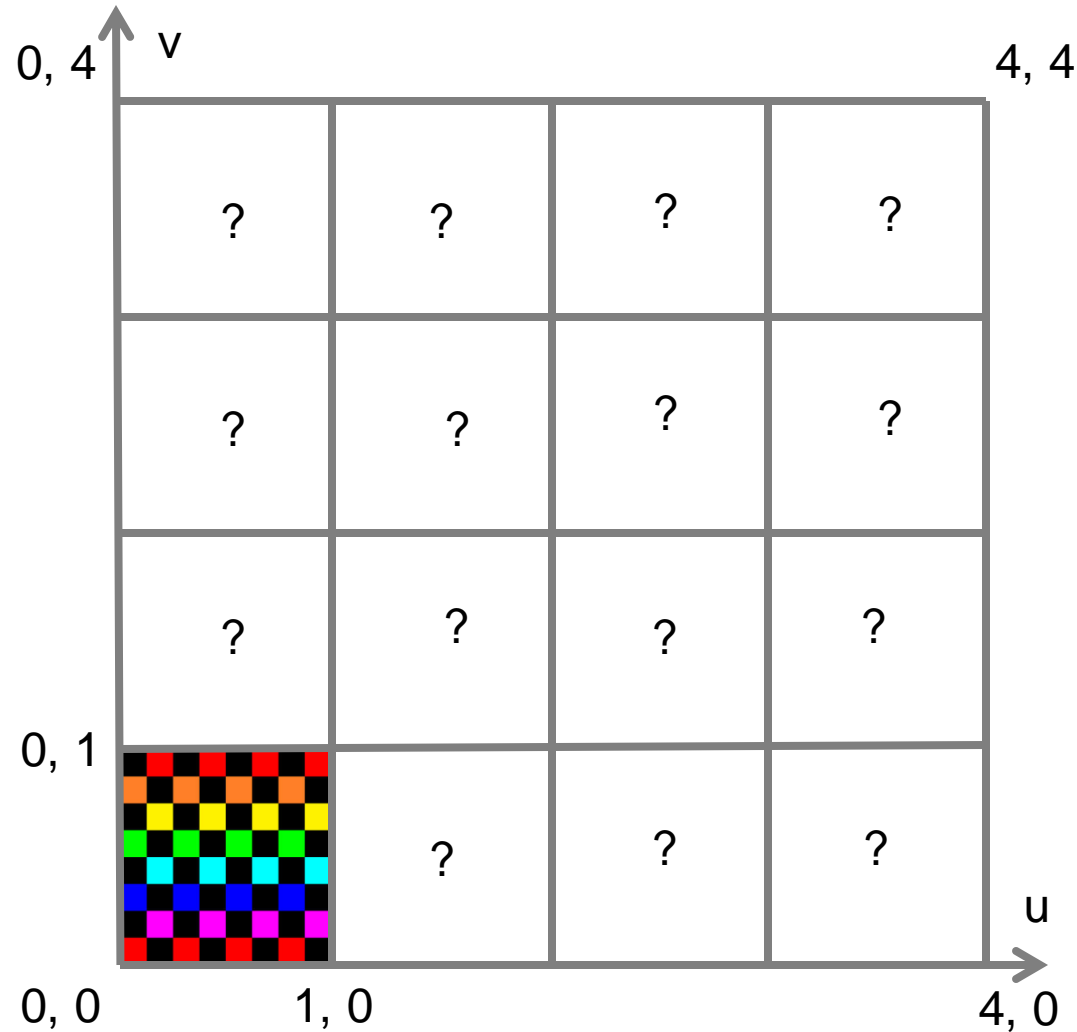
- **Image textures**

- Return the color of the image at a given point
- Point defined by mapping the texture coordinates $[0,1]^n$ to the entire image
- Images may be 1D (line of pixels), 2D, and 3D (stacks of images)
- Coordinates outside of $[0,1]^2$ can be mapped in different modes



Wrap Mode

- **Texture Coordinates**
 - (u, v) in $[0, 1] \times [0, 1]$
- **What if?**
 - (u, v) not in unit square?



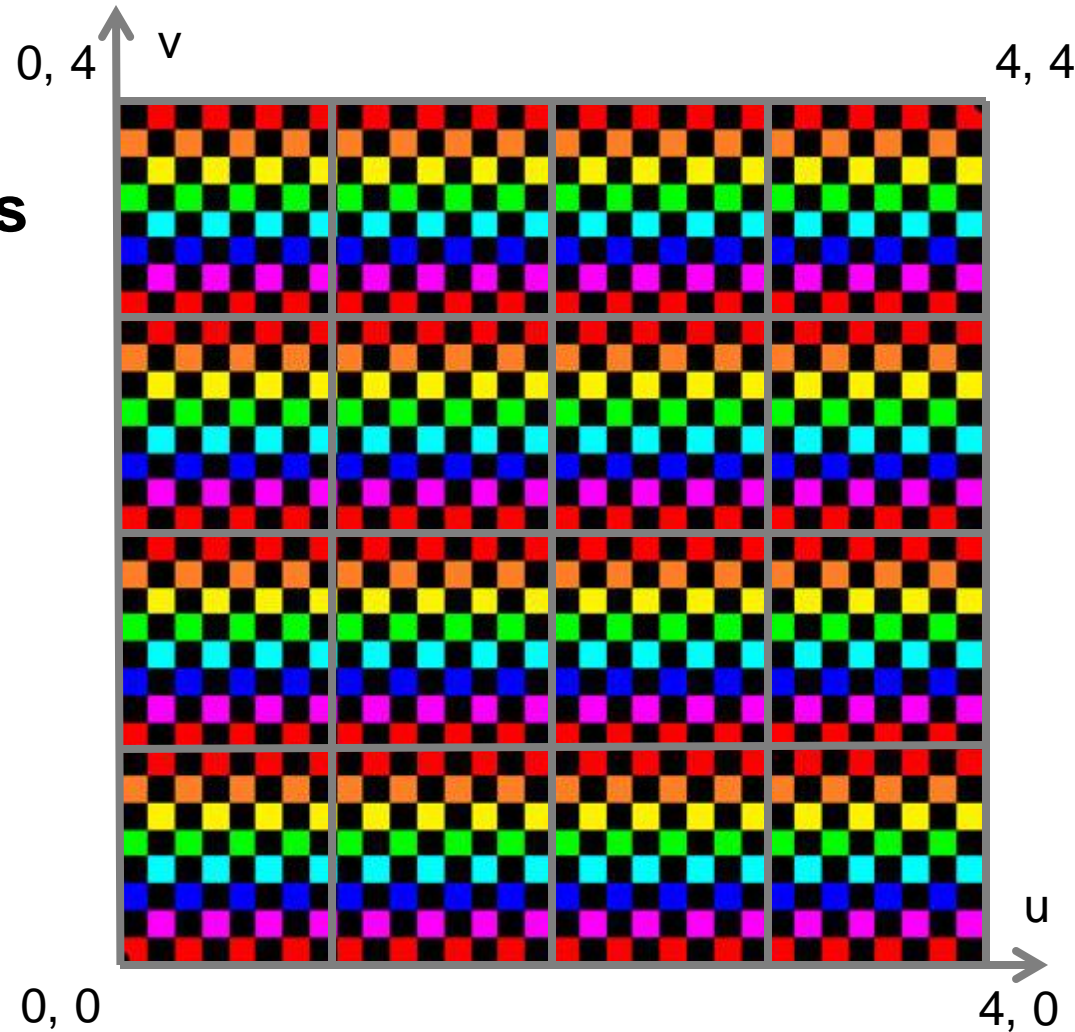
Wrap Mode

- Repeat

- Fractional Coordinates

- $t_u = u - \lfloor u \rfloor$

- $t_v = v - \lfloor v \rfloor$



Wrap Mode

- **Mirror**

- **Fractional Coordinates**

- $t_u = u - [u]$

- $t_v = v - [v]$

- **Lattice Coordinates**

- $l_u = [u]$

- $l_v = [v]$

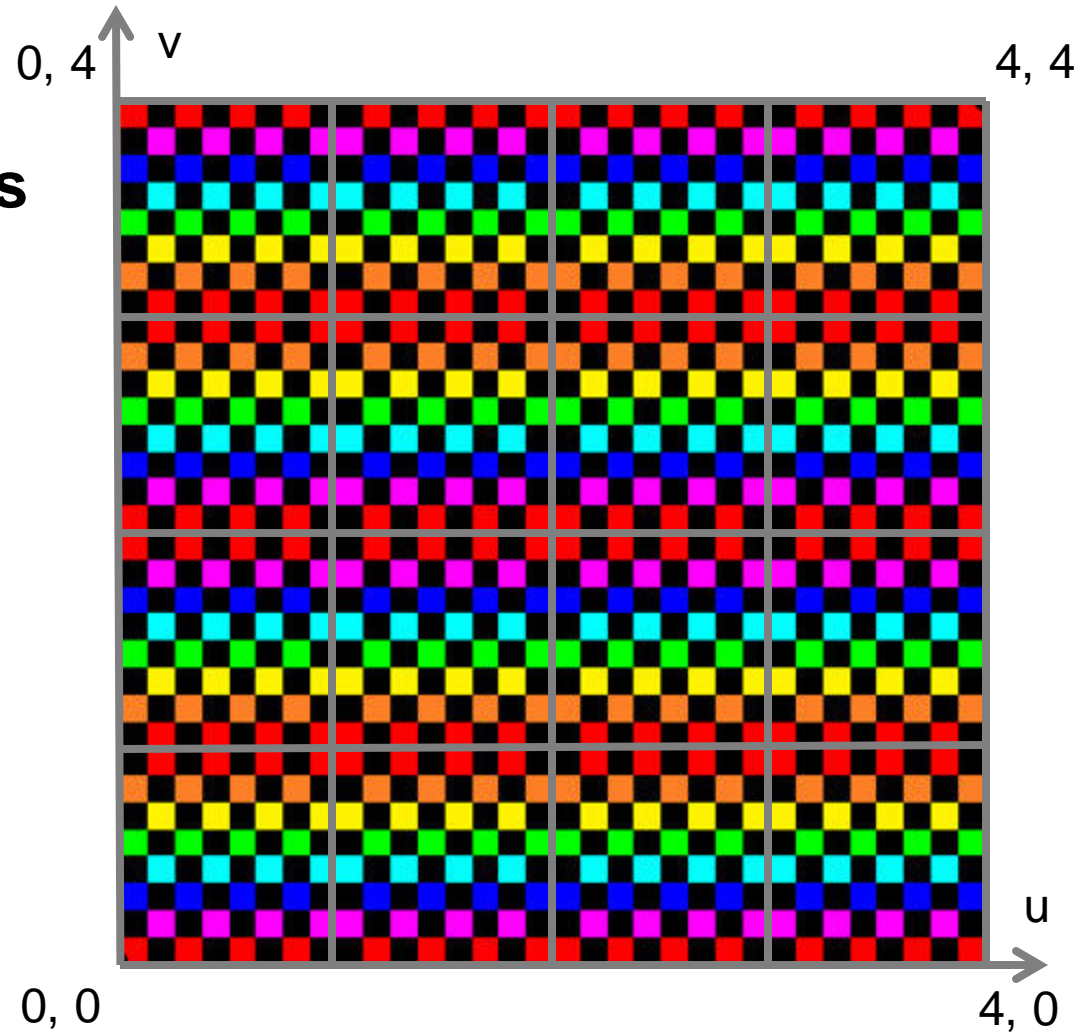
- **Mirror if Odd**

- if ($l_u \% 2 == 1$)

- $t_u = 1 - t_u$

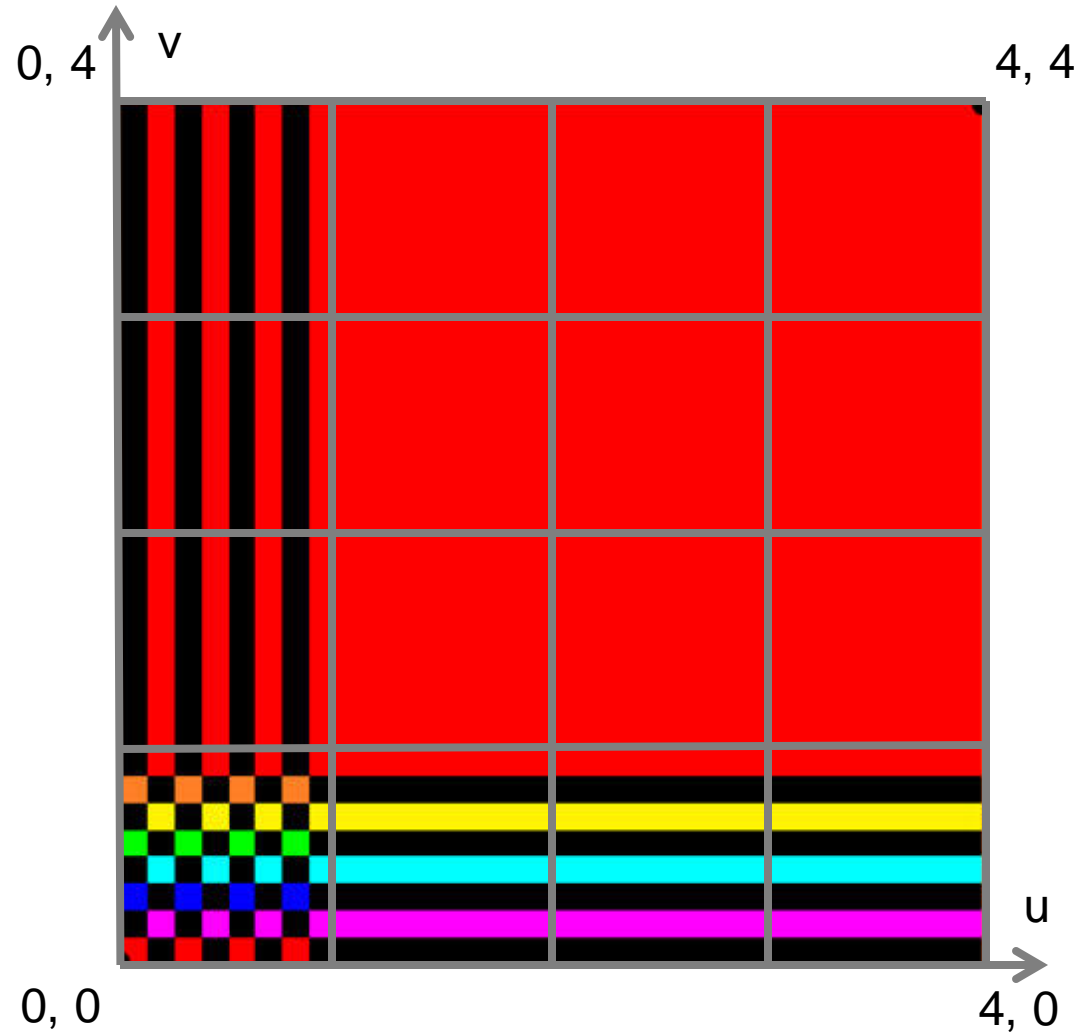
- if ($l_v \% 2 == 1$)

- $t_v = 1 - t_v$



Wrap Mode

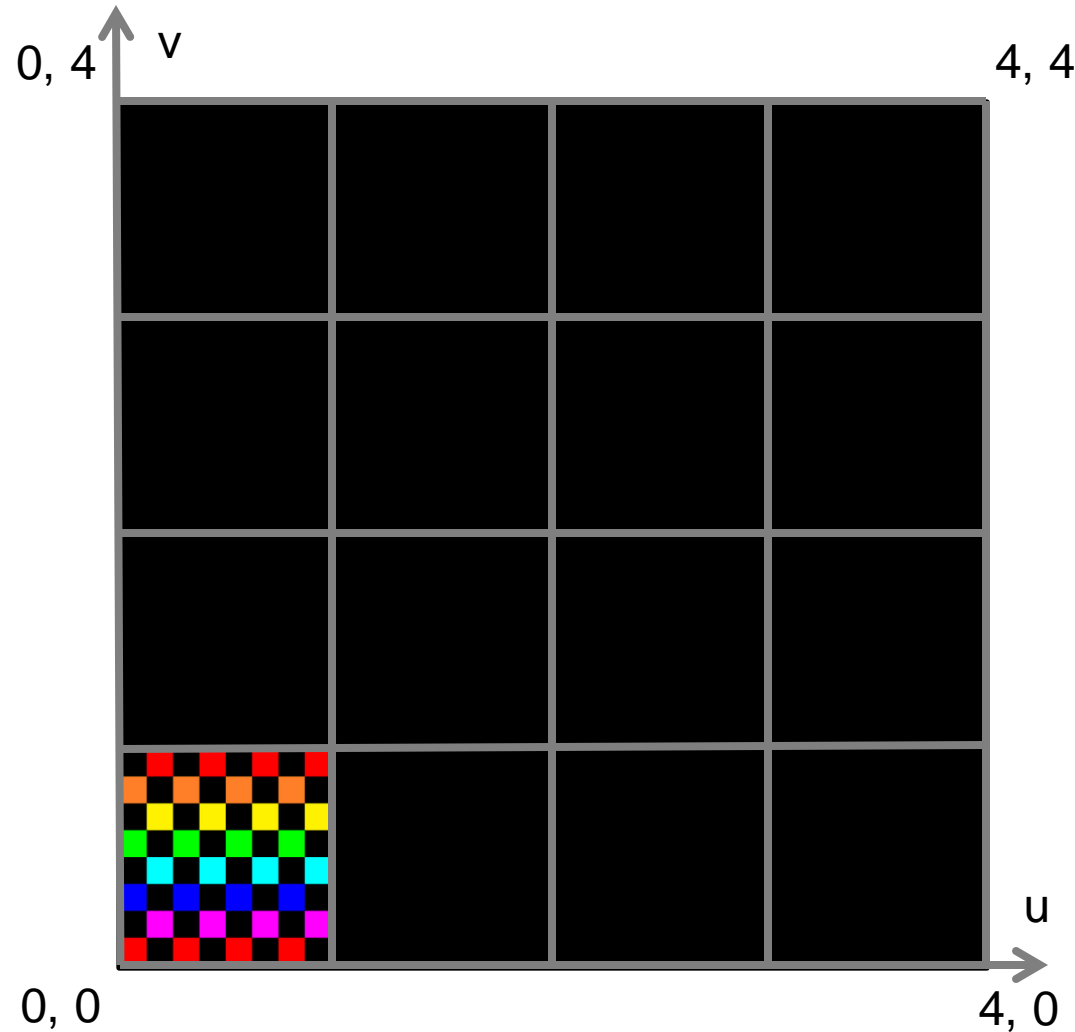
- **Clamp**
- **Clamp u to [0, 1]**
if $(u < 0)$ $tu = 0$;
else if $(u > 1)$ $tu = 1$;
else $tu = u$;
- **Clamp v to [0, 1]**
if $(v < 0)$ $tv = 0$;
else if $(v > 1)$ $tv = 1$;
else $tv = v$;



Wrap Mode

- **Border**
 - Border color can be explicitly defined
- **Check Bounds**

```
if (u < 0 || u > 1
    || v < 0 || v > 1)
    return backgroundColor;
else
    tu = u;
    tv = v;
```



Wrap Mode

- **Comparison**
 - With OpenGL texture modes



GL_REPEAT



GL_MIRRORED_REPEAT



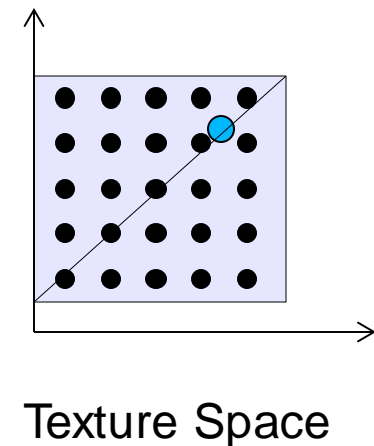
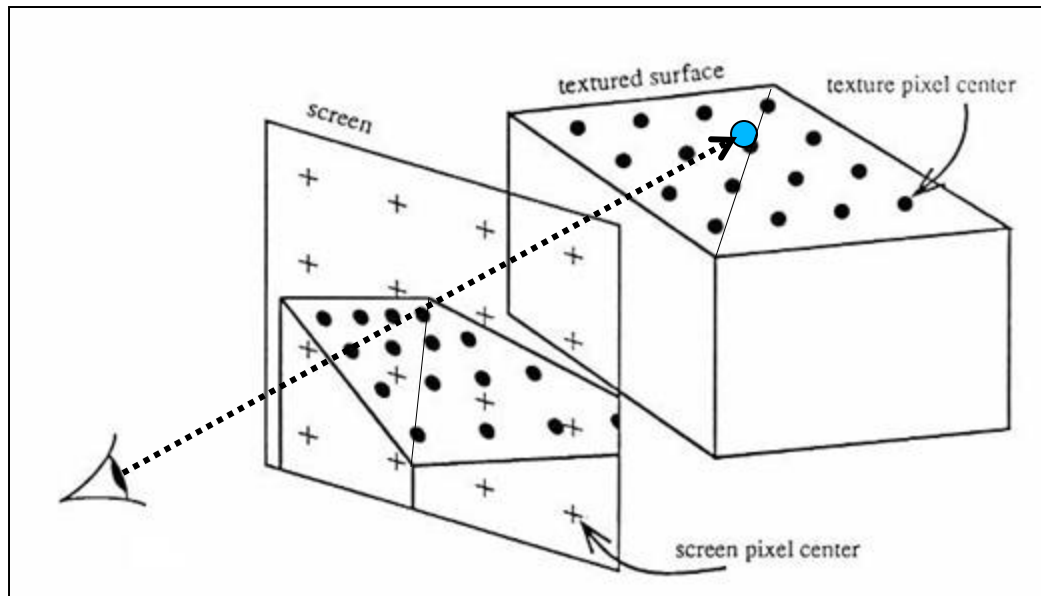
GL_CLAMP_TO_EDGE



GL_CLAMP_TO_BORDER

Reconstruction Filter

- **Image texture**
 - Discrete set of sample values (given at texel centers!)
- **In general**
 - Hit point does not exactly hit a texture sample
- **Still want to reconstruct a continuous function**
 - Use a *reconstruction filter* to find color for hit point



Nearest Neighbor

- **Local Coordinates**

- Assuming cell-centered samples
- $u = tu * \text{resU};$
- $v = tv * \text{resV};$

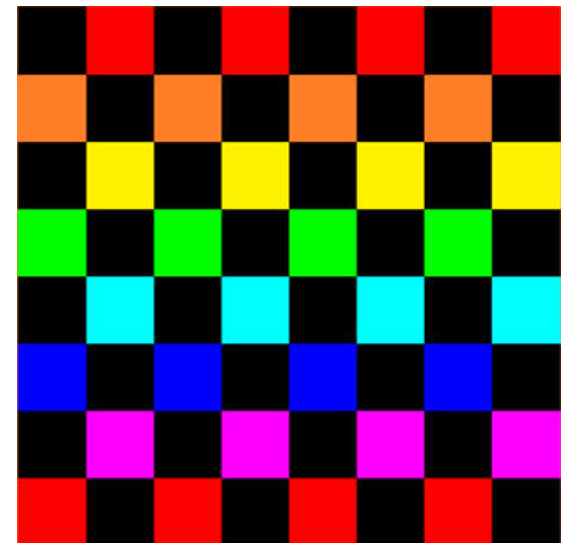
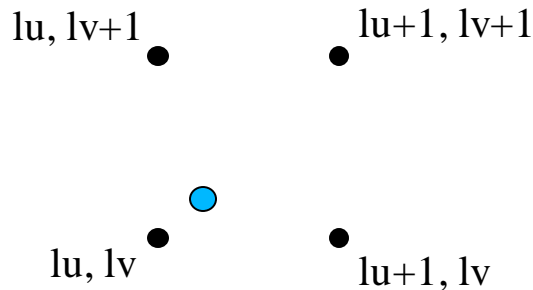
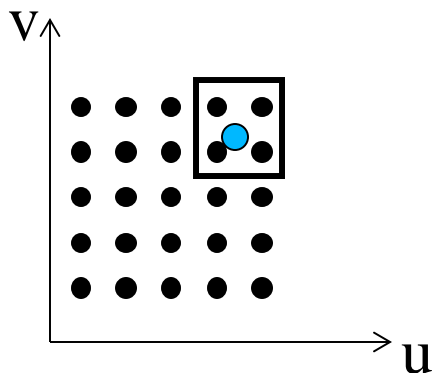
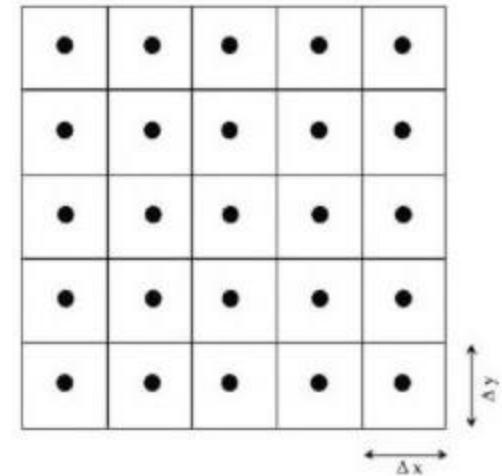
- **Lattice Coordinates**

- $lu = \min(\lfloor u \rfloor, \text{resU} - 1);$
- $lv = \min(\lfloor v \rfloor, \text{resV} - 1);$

- **Texture Value**

- return image[lu, lv];

Pixel centred registration



Bilinear Interpolation

- **Local Coordinates**

- Assuming node-centered samples
- $u = tu * (\text{resU} - 1);$
- $v = tv * (\text{resV} - 1);$

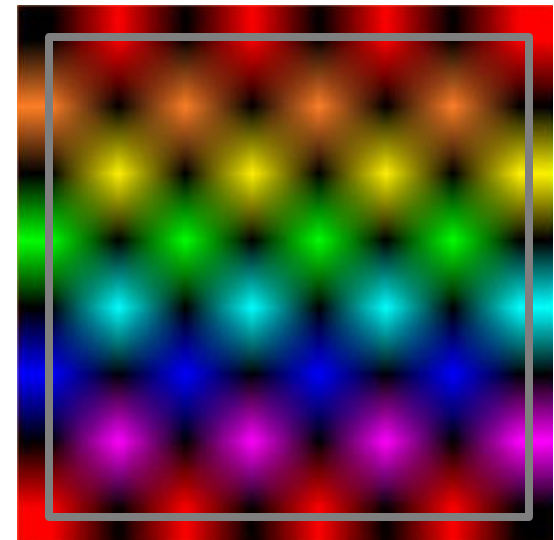
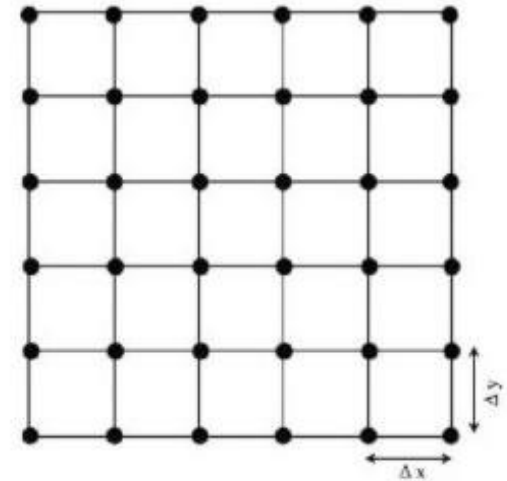
- **Fractional Coordinates**

- $fu = u - \lfloor u \rfloor;$
- $fv = v - \lfloor v \rfloor;$

- **Texture Value**

- $\text{return } (1-fu) (1-fv) \text{ image}[\lfloor u \rfloor, \lfloor v \rfloor]$
+ $(1-fu) (fv) \text{ image}[\lfloor u \rfloor, \lfloor v \rfloor + 1]$
+ $(fu) (1-fv) \text{ image}[\lfloor u \rfloor + 1, \lfloor v \rfloor]$
+ $(fu) (fv) \text{ image}[\lfloor u \rfloor + 1, \lfloor v \rfloor + 1]$

Grid node registration



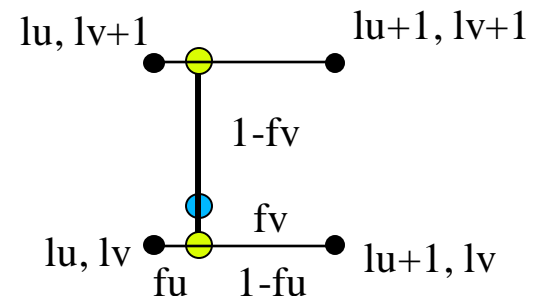
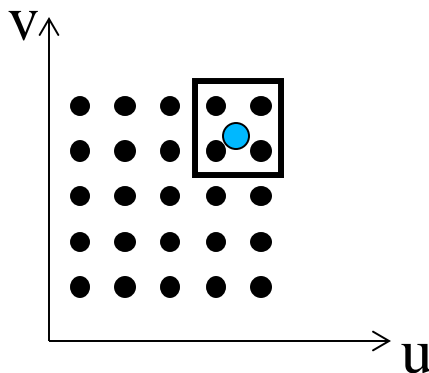
Bilinear Interpolation

- **Successive Linear Interpolations**

- $u0 = (1-fv) \text{ image}[\lfloor u \rfloor, \lfloor v \rfloor]$
+ $(fv) \text{ image}[\lfloor u \rfloor, \lfloor v \rfloor + 1];$

- $u1 = (1-fv) \text{ image}[\lfloor u \rfloor + 1, \lfloor v \rfloor]$
+ $(fv) \text{ image}[\lfloor u \rfloor + 1, \lfloor v \rfloor + 1];$

- return $(1-fu) u0$
+ $(fu) u1;$



Nearest vs. Bilinear Interpolation



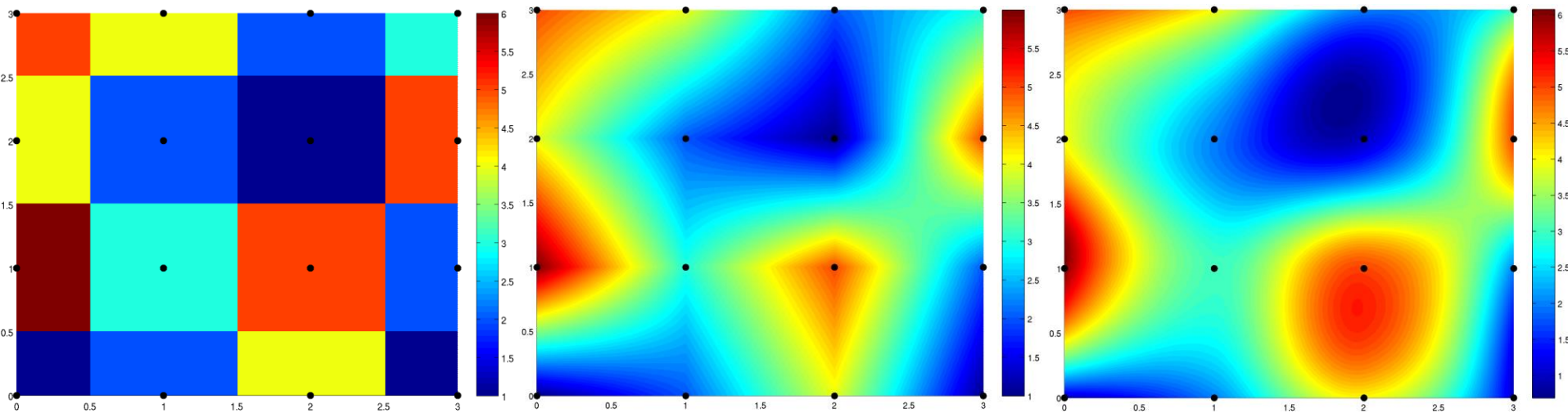
GL_NEAREST



GL_LINEAR

Bicubic Interpolation

- **Properties**
 - Assuming node-centered samples
 - Essentially based on cubic splines (see later)
- **Pros**
 - Even smoother
- **Cons**
 - More complex & expensive (4x4 kernel)
 - Overshoot



Discussion: Image Textures

- **Pros**

- Simple generation
 - Painted, simulation, ...
- Simple acquisition
 - Photos, videos

- **Cons**

- Illumination “frozen” during acquisition (e.g. photo)
 - Limited resolution
 - Susceptible to aliasing
 - High memory requirements (often HUGE for films, 100s of GB)
 - Issues when mapping 2D image onto 3D object
-

PROCEDURAL TEXTURES

Discussion: Procedural Textures

- **Cons**

- Sometimes hard to achieve specific effect
- Possibly non-trivial programming

- **Pros**

- Flexibility & parametric control
 - Unlimited resolution
 - Anti-aliasing possible
 - Low memory requirements
 - May be directly defined as 3D “image” mapped to 3D geometry
 - High visual complexity with low-cost
-

2D Checkerboard Function

- **Lattice Coordinates**

- $l_u = \lfloor u \rfloor$

- $l_v = \lfloor v \rfloor$

- **Compute Parity**

- $\text{parity} = (l_u + l_v) \% 2;$

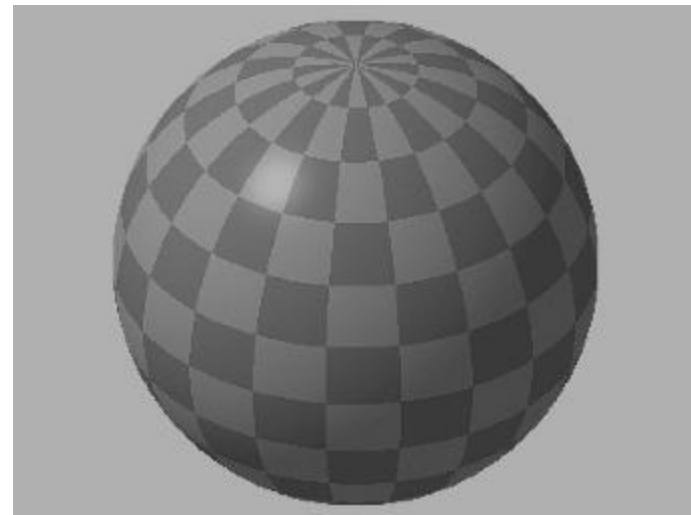
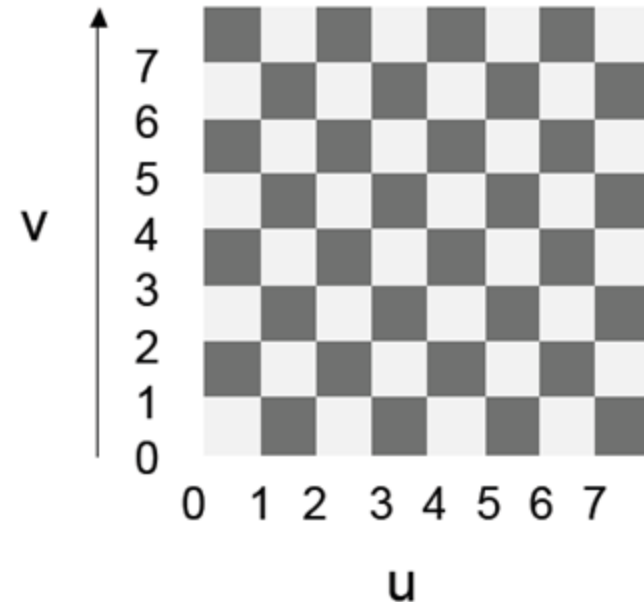
- **Return Color**

- if ($\text{parity} == 1$)

- return color1;

- else

- return color0;



3D Checkerboard - Solid Texture

- **Lattice Coordinates**

- $l_u = \lfloor u \rfloor$

- $l_v = \lfloor v \rfloor$

- $l_w = \lfloor w \rfloor$

- **Compute Parity**

- $\text{parity} = (l_u + l_v + l_w) \% 2;$

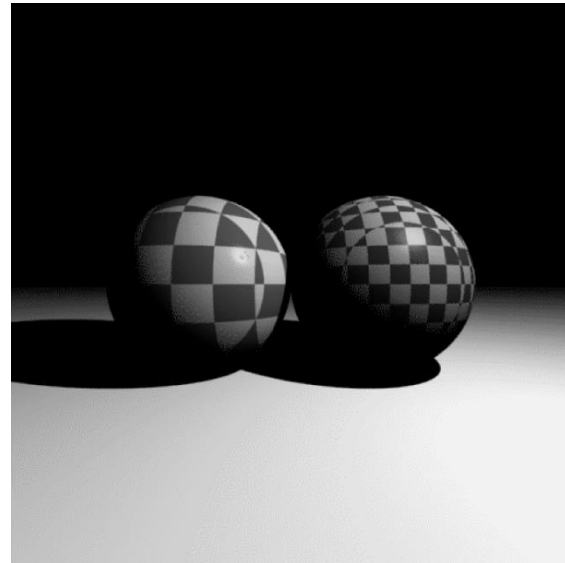
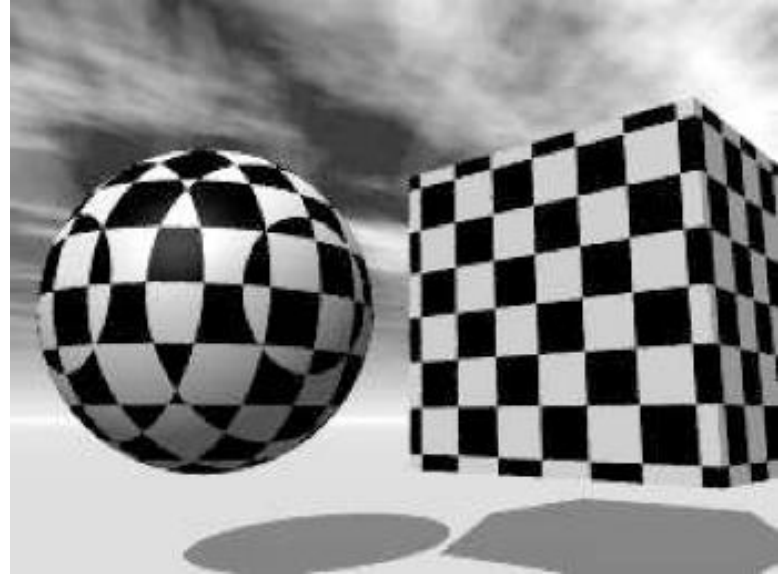
- **Return Color**

- if ($\text{parity} == 1$)

- return color1;

- else

- return color0;



Tile

- **Fractional Coordinates**

- $f_u = u - \lfloor u \rfloor$

- $f_v = v - \lfloor v \rfloor$

- **Compute Booleans**

- $b_u = f_u < \text{mortarWidth};$

- $b_v = f_v < \text{mortarWidth};$

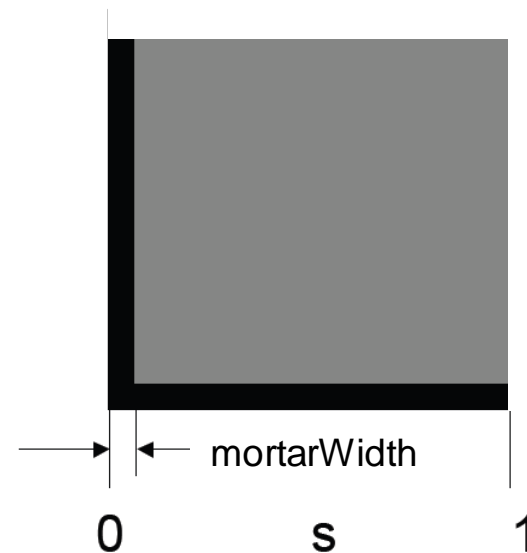
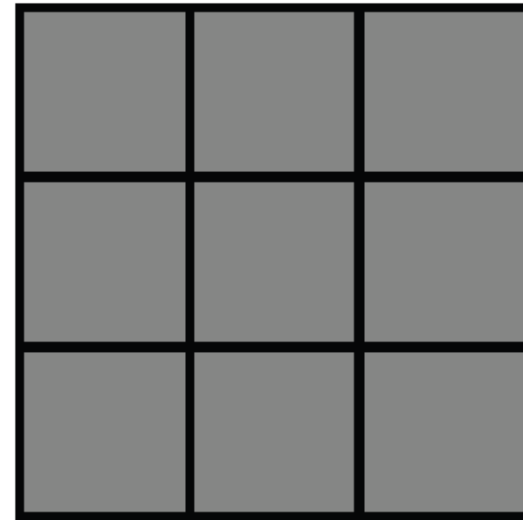
- **Return Color**

- if ($b_u \parallel b_v$)

- return mortarColor;

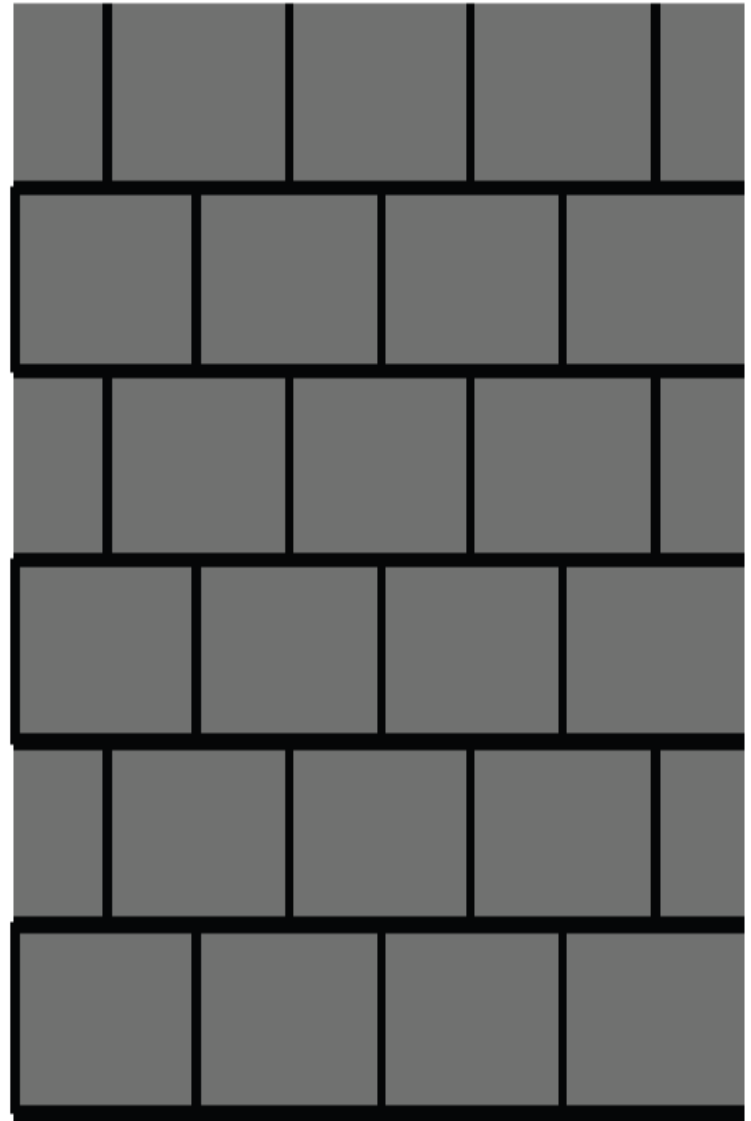
- else

- return tileColor;

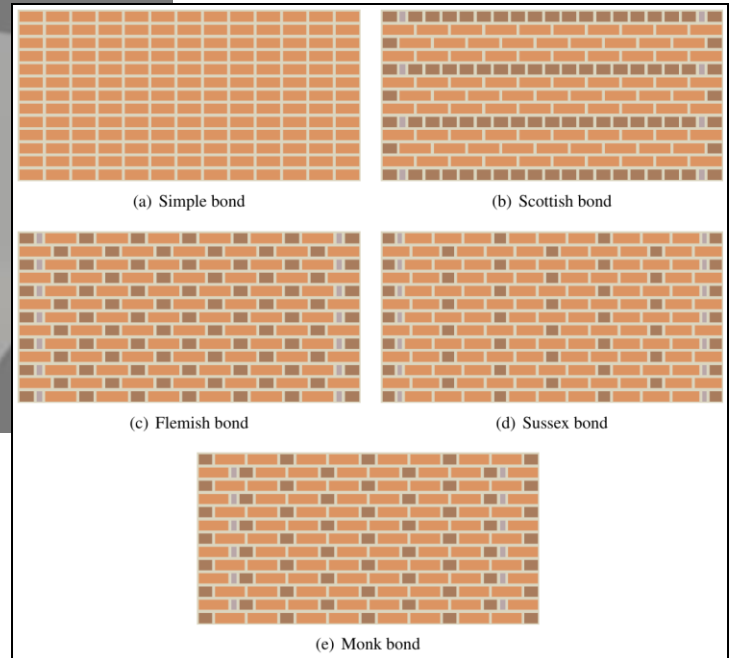
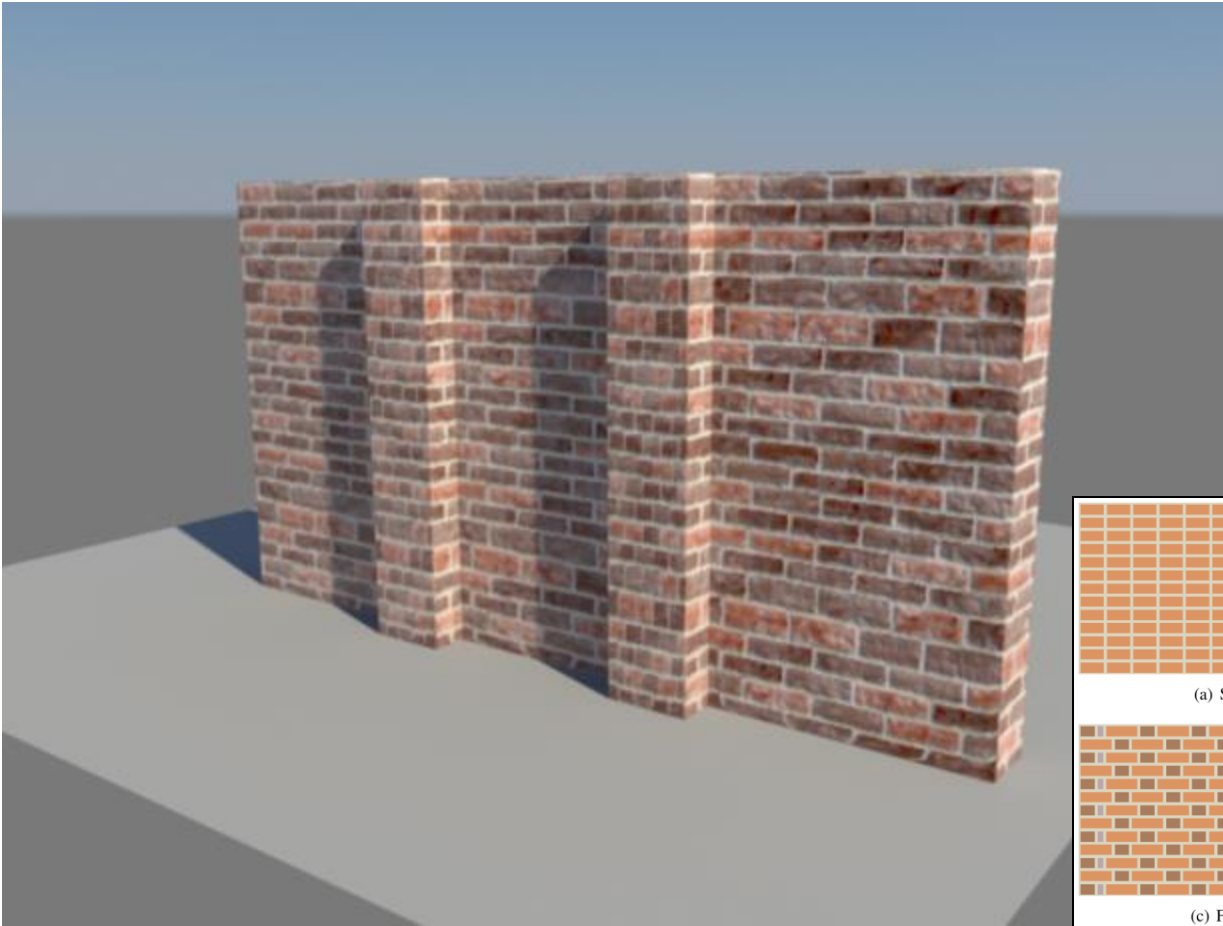


Brick

- **Shift Column for Odd Rows**
 - $\text{parity} = \lfloor v \rfloor \% 2;$
 - $u -= \text{parity} * 0.5;$
- **Fractional Coordinates**
 - $f_u = u - \lfloor u \rfloor$
 - $f_v = v - \lfloor v \rfloor$
- **Compute Booleans**
 - $bu = f_u < \text{mortarWidth};$
 - $bv = f_v < \text{mortarWidth};$
- **Return Color**
 - if ($bu \parallel bv$)
 - return mortarColor;
 - else
 - return brickColor;

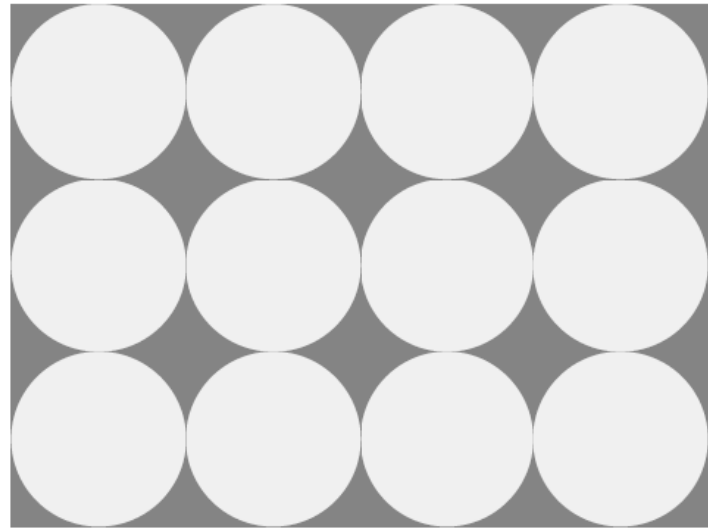


More Variation

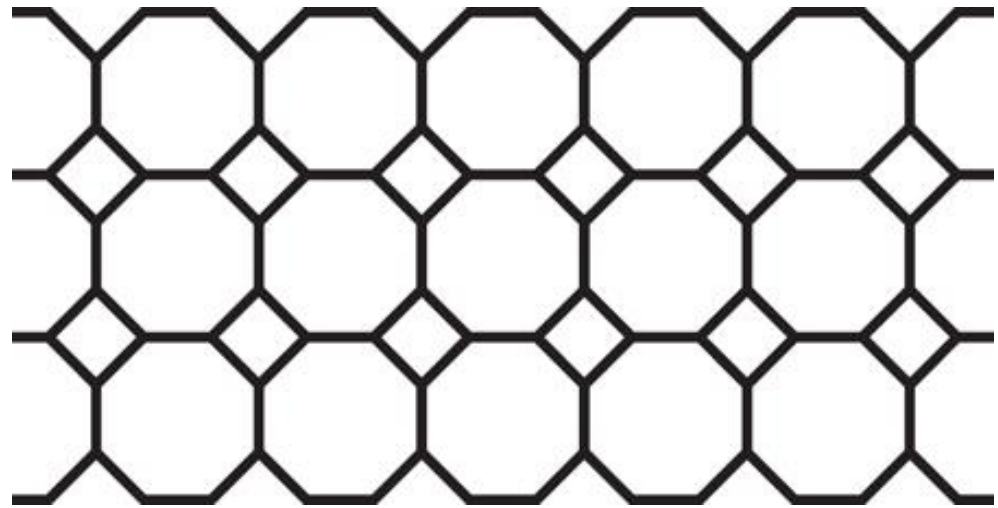


Other Patterns

- **Circular Tiles**



- **Octagonal Tiles**



- **Use your imagination!**
-

Perlin Noise

- **Natural Patterns**

- Similarity between patches at different locations
 - Repetitiveness, coherence (e.g., skin of a tiger or zebra)
- Similarity on different resolution scales
 - Self-similarity
- But never completely identical
 - Additional disturbances, turbulence, noise

- **Mimic Statistical Properties**

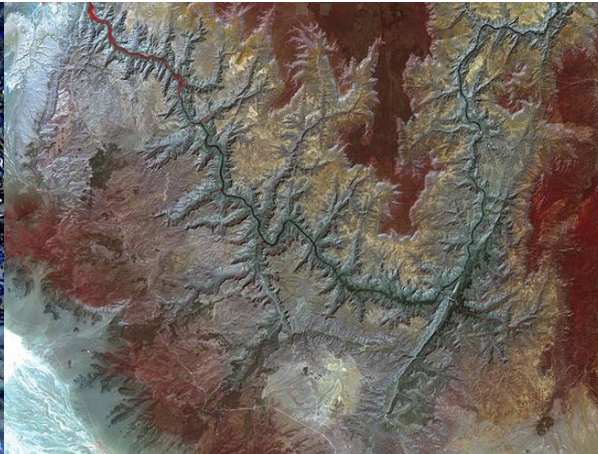
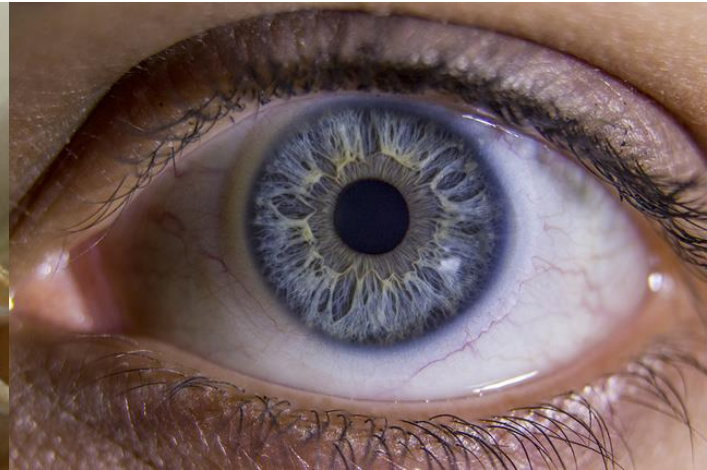
- Purely empirical approach
- Looks convincing, but has nothing to do with material's physics

- **Perlin Noise is essential for adding “natural” details**

- Used in many texture functions
-

Perlin Noise

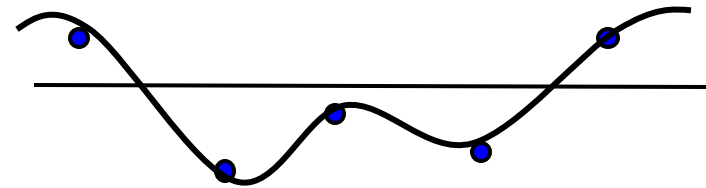
- Natural Fractals



Noise Function

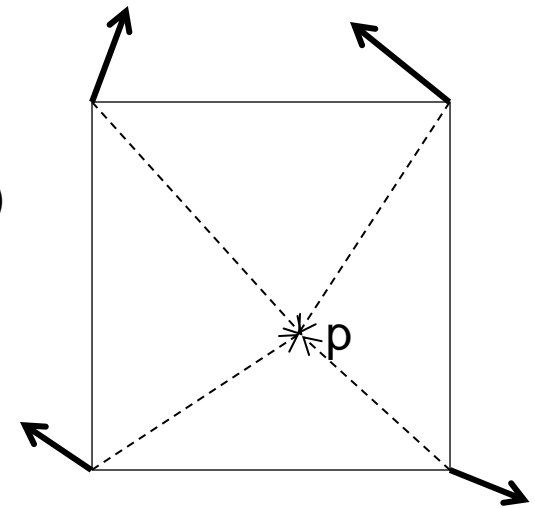
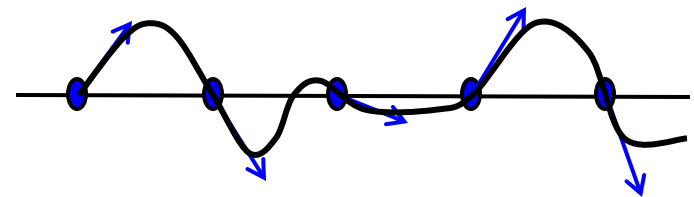
- **Noise(x, y, z) Function**

- Statistical invariance under rotation
- Statistical invariance under translation
- Roughly fixed frequency of ~ 1 Hz



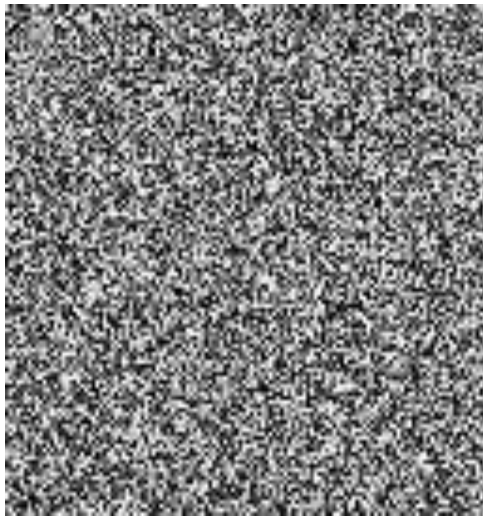
- **Integer Lattice (i, j, k)**

- Value noise
 - Random value at lattice points
- Gradient noise (most common)
 - Random gradient vector at lattice point
- Interpolation
 - Bi-/tri-linear or cubic (Hermite spline, \rightarrow later)
- Hash function to map vertices to values
 - Essentially randomized look up
 - Virtually infinite extent and variation with finite array of values

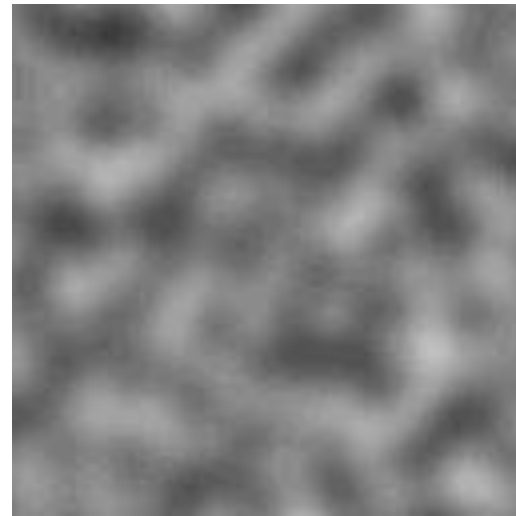


Noise vs. Noise

- **Value Noise vs. Gradient Noise**
 - Gradient noise has lower regularity artifacts
 - More high frequencies in noise spectrum
- **Random Values vs. Perlin Noise**
 - Stochastic vs. deterministic



Random values
at each pixel



Gradient noise

Turbulence Function

- **Noise Function**

- Single spike in frequency spectrum (single frequency, see later)

- **Natural Textures**

- Mix of different frequencies
- Decreasing amplitude for high frequencies

- **Turbulence from Noise**

- $Turbulence(x) = \sum_{i=0}^k |a_i * noise(f_i x)|$

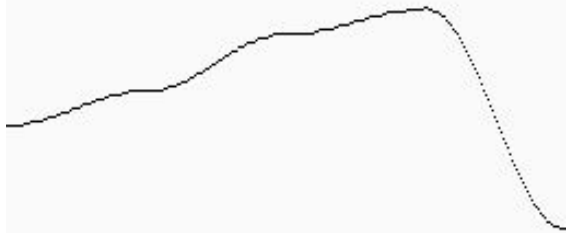
- Frequency: $f_i = 2^i$
- Amplitude: $a_i = 1 / p^i$
- Persistence: p typically $p=2$
- Power spectrum : $a_i = 1 / f_i$
- Brownian motion: $a_i = 1 / f_i^2$

- Summation truncation

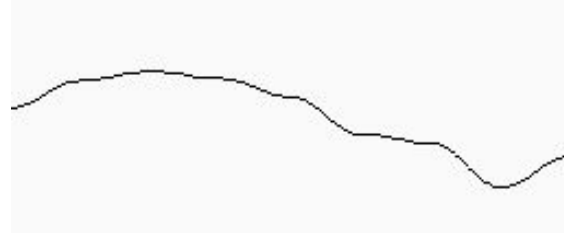
- 1st term: noise(x)
 - 2nd term: noise(2x)/2
 - ...
 - Until period $(1/f_k) < 2$ pixel-size (band limit, see later)
-

Synthesis of Turbulence (1-D)

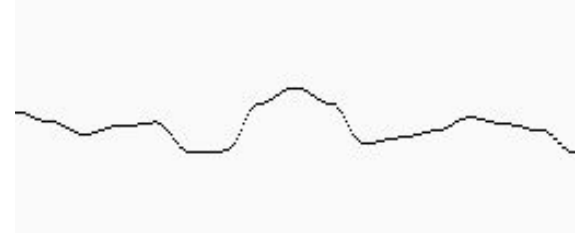
Amplitude : 128
frequency : 4



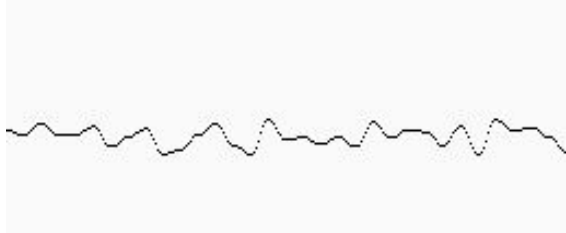
Amplitude : 64
frequency : 8



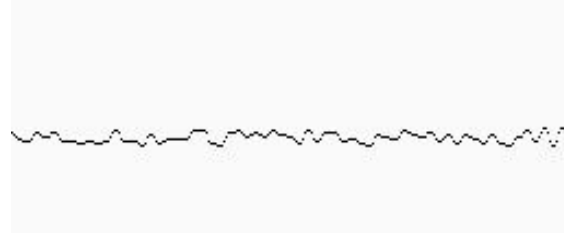
Amplitude : 32
frequency : 16



Amplitude : 16
frequency : 32



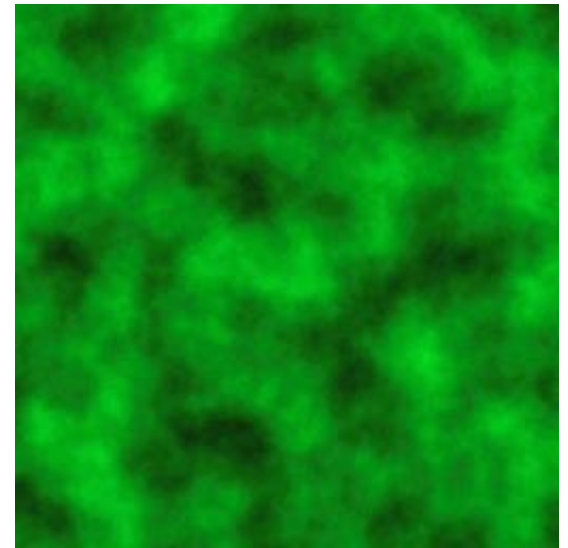
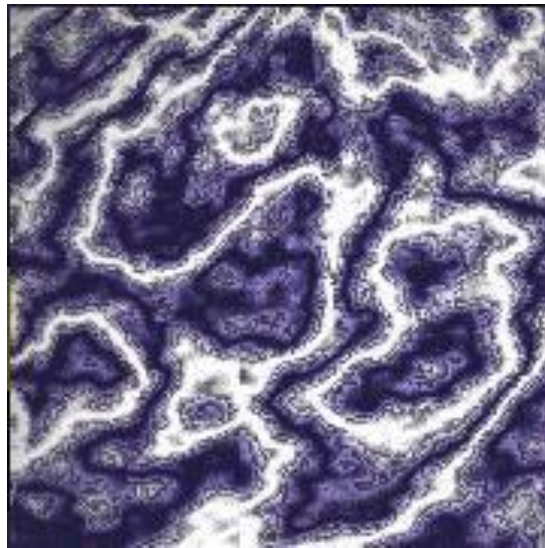
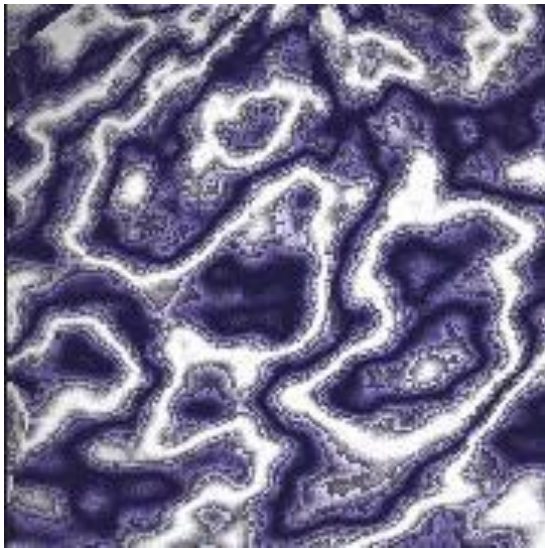
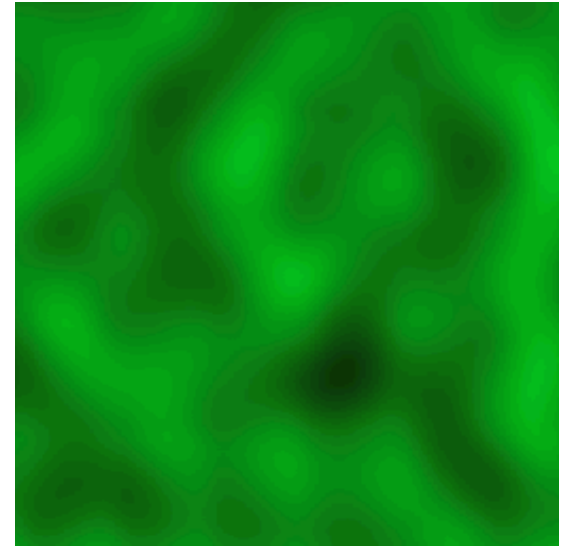
Amplitude : 8
frequency : 64



Sum of Noise Functions = (Perlin Noise)



Synthesis of Turbulence (2-D)



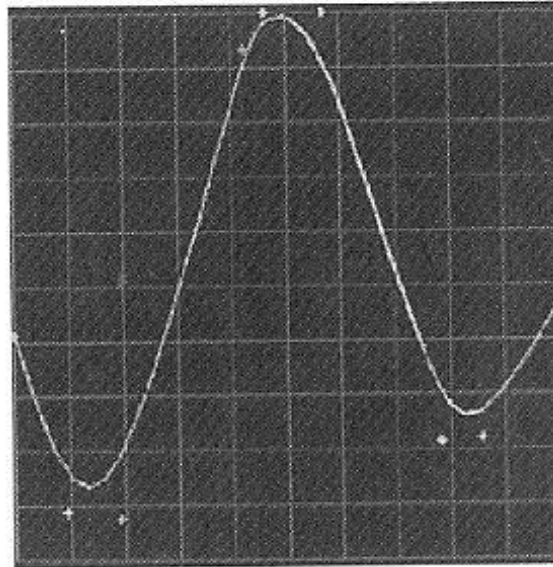
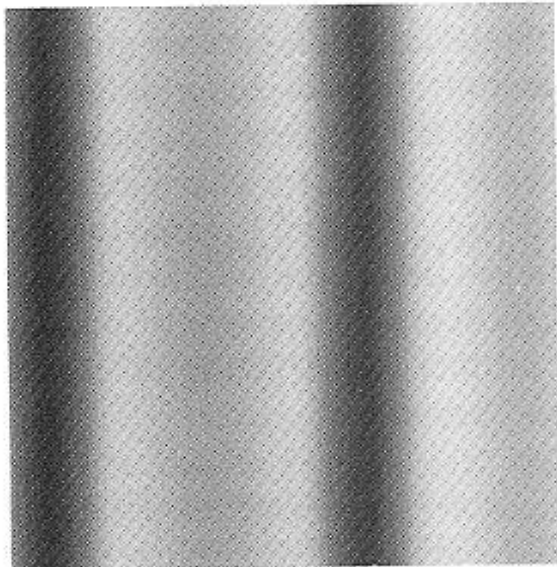
Example: Marble

- **Overall Structure**

- Smoothly alternating layers of different marble colors
- $f_{\text{marble}}(x,y,z) := \text{marble_color}(\sin(x))$
- `marble_color` : transfer function (see lower left)

- **Realistic Appearance**

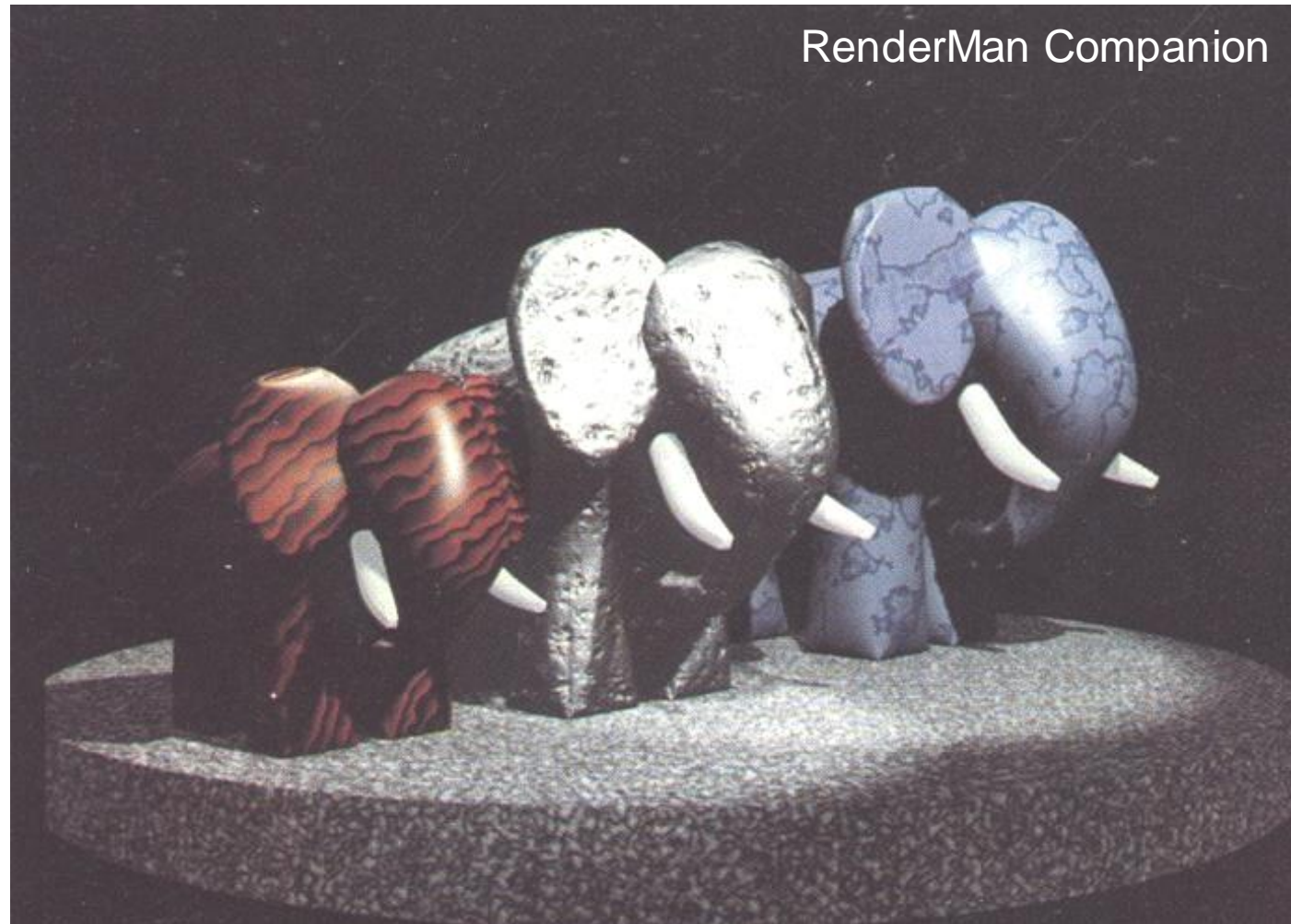
- Simulated turbulence
- $f_{\text{marble}}(x,y,z) := \text{marble_color}(\sin(x + \text{turbulence}(x, y, z)))$



Solid Noise

- **3D Noise Texture**

- Wood
- Erosion
- Marble
- Granite
- ...



Others Applications

- **Bark**
 - Turbulated saw-tooth function

 - **Clouds**
 - White blobs
 - Turbulated transparency along edge

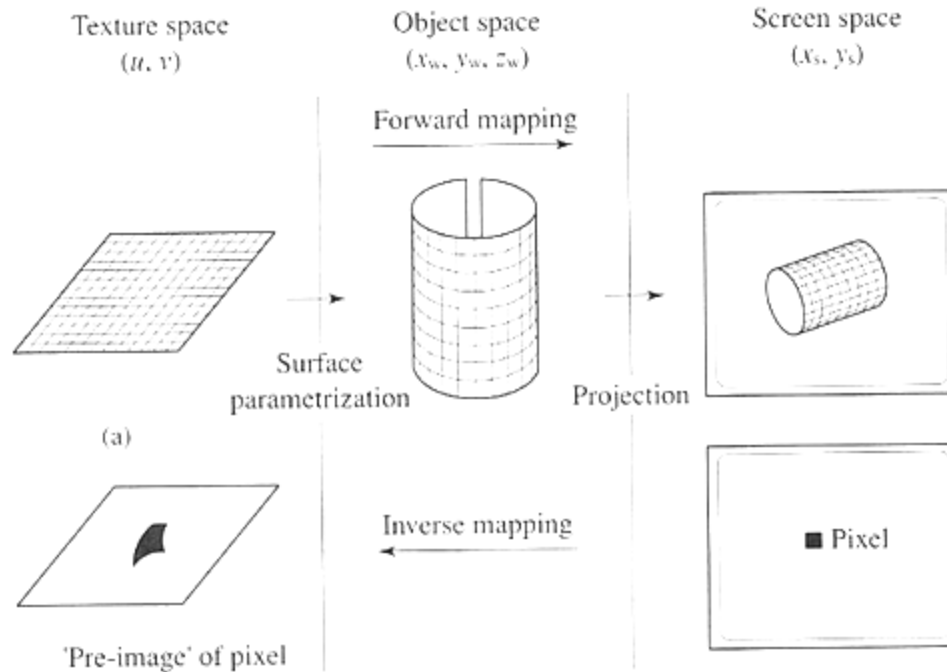
 - **Animation**
 - Vary procedural texture function's parameters over time
-

Shading Languages

- **Small program fragments (plugins)**
 - Compute certain aspects of the rendering process
 - Executing at innermost loop, must be extremely efficient
 - Executed at each intersection
- **Typical shaders**
 - Material/surface shaders: Compute reflected color
 - Light shaders: Compute illumination from light source at some point
 - Volume shader: Compute interaction in participating medium
 - Displacement shader: Compute changes to the geometry
 - Camera shader: Compute rays for each pixel
- **Shading languages**
 - RenderMan (the mother of all shading languages)
 - HLSL (DX only), GLSL (OpenGL only), CG (Nvidia only)
 - Currently no portable shading format usable for exchange
 - But Material Definition Language (MDL, Nvidia), shade.js (UdS)
- **More details later**

TEXTURE MAPPING

2D Texture Mapping



- **Forward mapping**

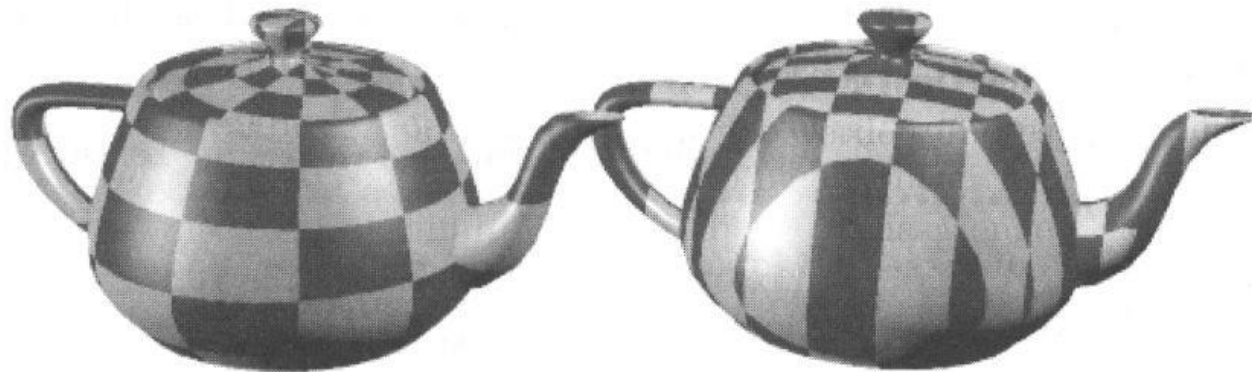
- Object surface parameterization, plus
- Projective transformation to the screen

- **Inverse mapping**

- Find corresponding pre-image/footprint of each pixel in texture
- Integrate over pre-image

Surface Parameterization

- **To apply textures we need 2D coordinates on surfaces**
 - **Parameterization**
- **Some objects have a natural parameterization**
 - Sphere: spherical coordinates $(\varphi, \theta) = (2\pi u, \pi v)$
 - Cylinder: cylindrical coordinates $(\varphi, h) = (2\pi u, H v)$
 - Parametric surfaces (such as B-spline or Bezier surfaces → later)
- **Parameterization is less obvious for**
 - Polygons, implicit surfaces, teapots, ...



Triangle Parameterization

- **Triangle is a planar object**
 - Has implicit parameterization (e.g., barycentric coordinates)
 - But we need more control: Placement of triangle in texture space
- **Assign texture coordinates (u,v) to each vertex (x_o, y_o, z_o)**
- **Apply viewing projection $(x_o, y_o, z_o) \rightarrow (x,y)$ (details later)**
- **Yields full texture transformation (warping) $(u,v) \rightarrow (x,y)$**

$$x = \frac{au + bv + c}{gu + hv + i} \qquad y = \frac{du + ev + f}{gu + hv + i}$$

- In homogeneous coordinates (by embedding (u,v) as $(u,v,1)$)

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}; (x, y) = \left(\frac{x'}{w}, \frac{y'}{w} \right), (u, v) = \left(\frac{u'}{q}, \frac{v'}{q} \right)$$

- Transformation coefficients determined by 3 pairs $(u,v) \rightarrow (x,y)$
 - Three linear equations
 - Invertible iff neither set of points is collinear

Triangle Parameterization (2)

- **Given**
$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} u' \\ v' \\ q \end{bmatrix}$$

- **The inverse transform $(x,y) \rightarrow (u,v)$ is**

$$\begin{bmatrix} u' \\ v' \\ q \end{bmatrix} = \begin{bmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{bmatrix} \begin{bmatrix} x' \\ y' \\ w \end{bmatrix}$$

- **Coefficients must be calculated for each triangle**

- Rasterization

- Incremental bilinear update of (u', v', q) in screen space
- Using the partial derivatives of the linear function (i.e., constants)

- Ray tracing

- Evaluated at every intersection (via barycentric coordinates)

- **Often (partial) derivatives are needed as well**

- Explicitly given in matrix (colored for $\partial u / \partial x$, $\partial v / \partial x$, $\partial q / \partial x$)

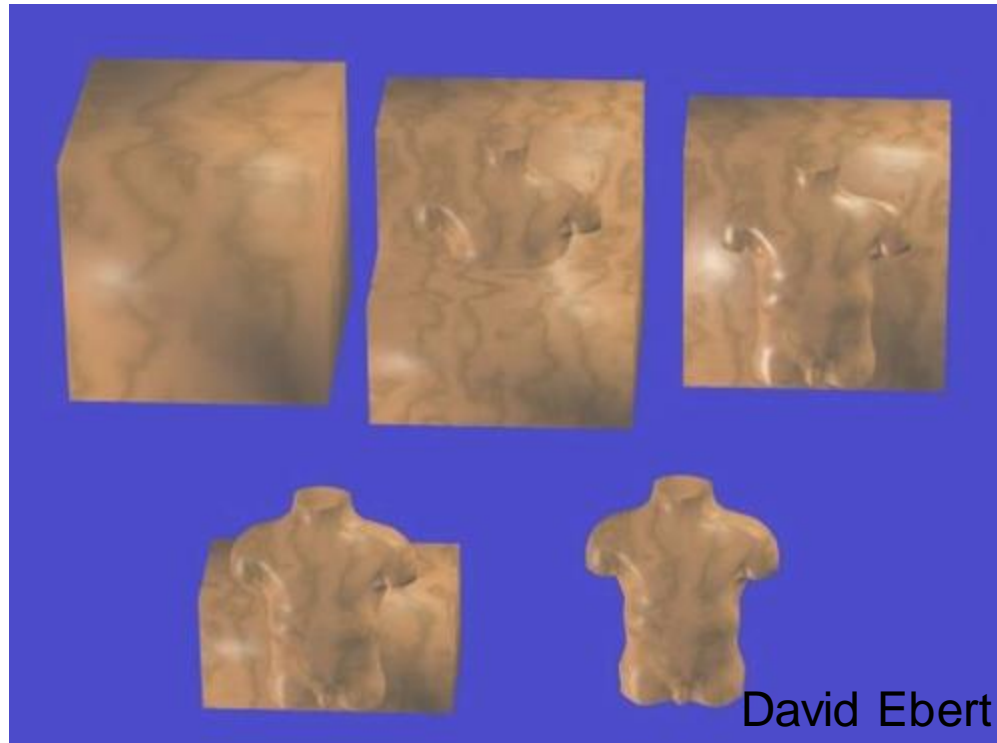
Textures Coordinates

- **Solid Textures**

- 3D world/object (x,y,z) coords \rightarrow 3D (u,v,w) texture coordinates
- Similar to carving object out of material block

- **2D Textures**

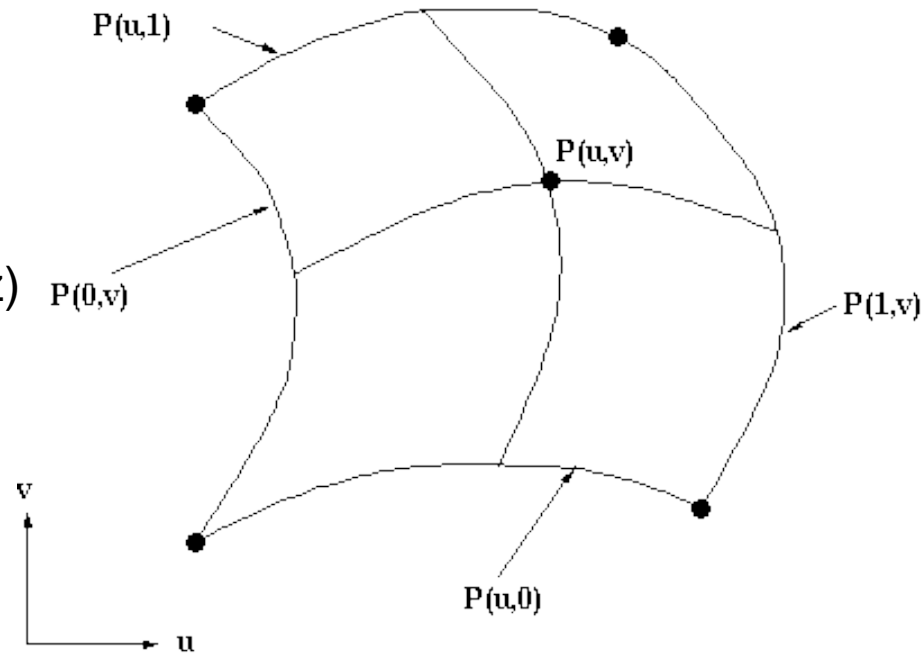
- 3D Cartesian (x,y,z) coordinates \rightarrow 2D (u,v) texture coordinates?



Parametric Surfaces

- **Definition (more detail later)**

- Surface defined by parametric function
 - $(x, y, z) = p(u, v)$
- Input
 - Parametric coordinates: (u, v)
- Output
 - Cartesian coordinates: (x, y, z)



- **Texture Coordinates**

- Directly derived from surface parameterization
 - Invert parametric function
 - From world coordinates to parametric coordinates
 - Usually computed implicitly anyway (e.g. in ray tracing)
-

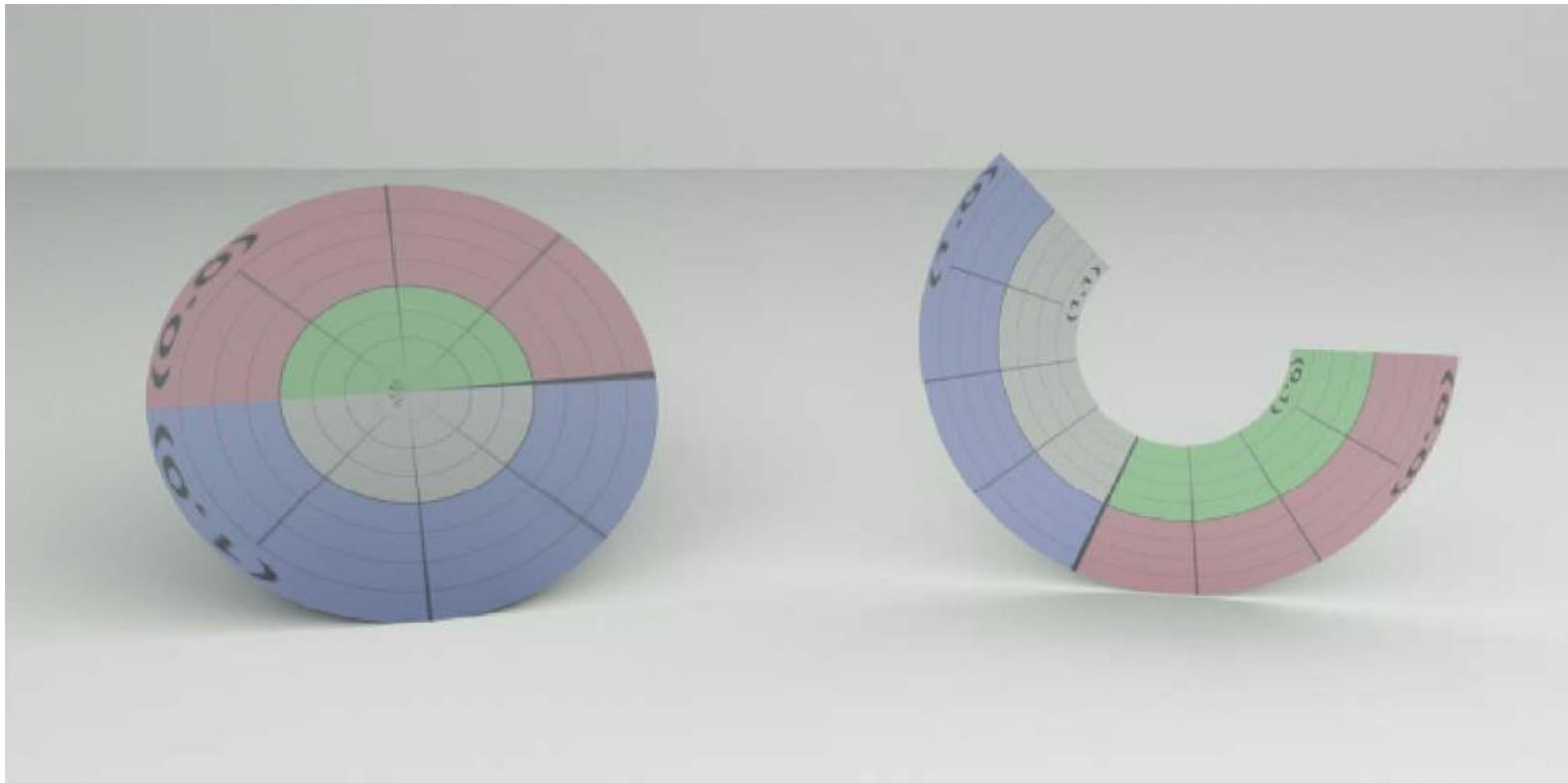
Parametric Surfaces

- **Polar Coordinates**

- $(x, y, 0) = \text{Polar2Cartesian}(r, \varphi)$

- **Disc**

- $p(u, v) = \text{Polar2Cartesian}(R v, 2 \pi u)$ // disc radius R



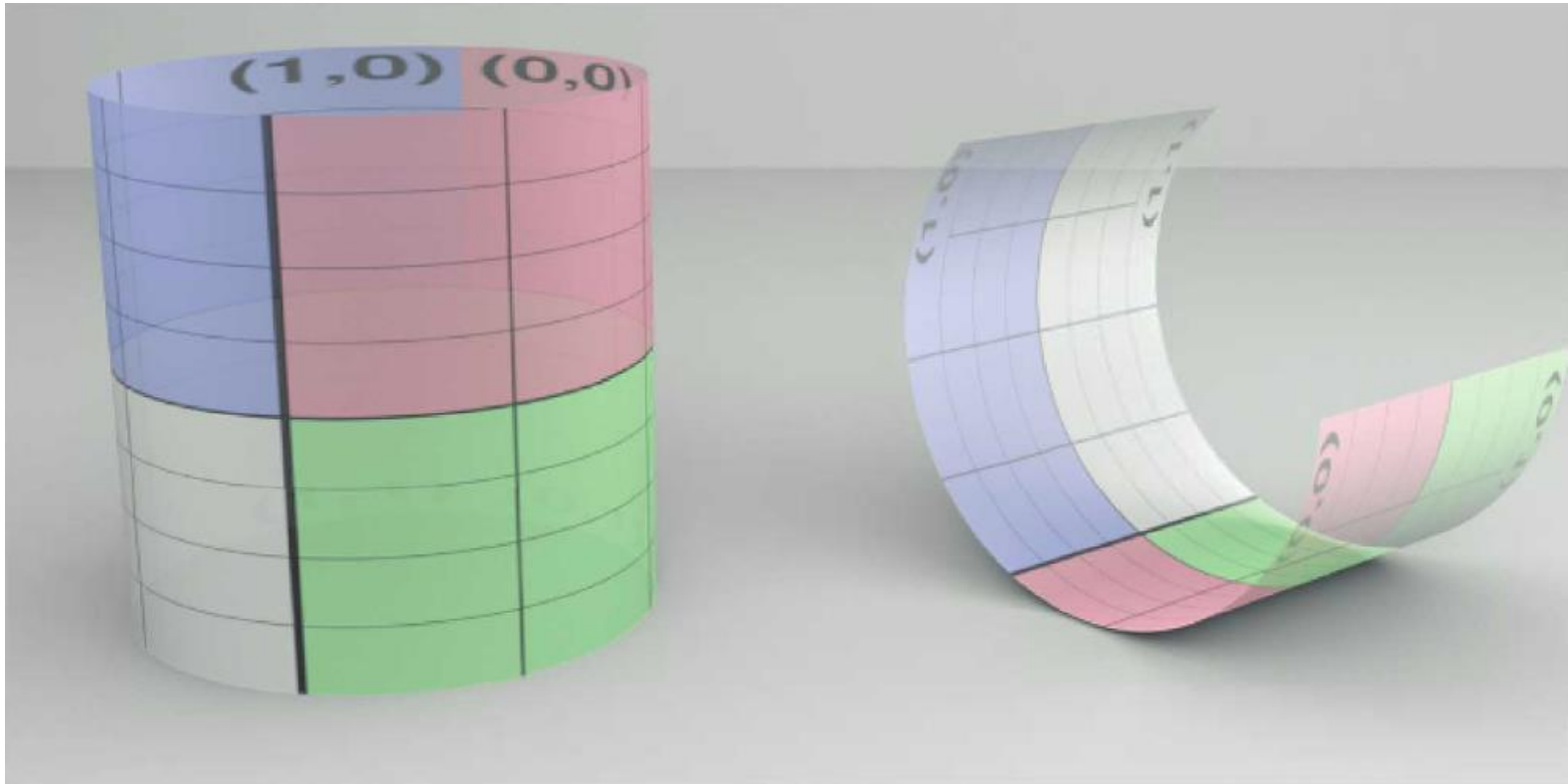
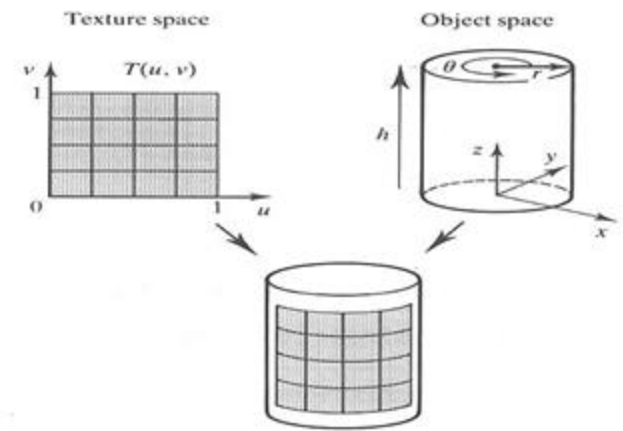
Parametric Surfaces

- **Cylindrical Coordinates**

- $(x, y, z) = \text{Cylindrical2Cartesian}(r, \varphi, z)$

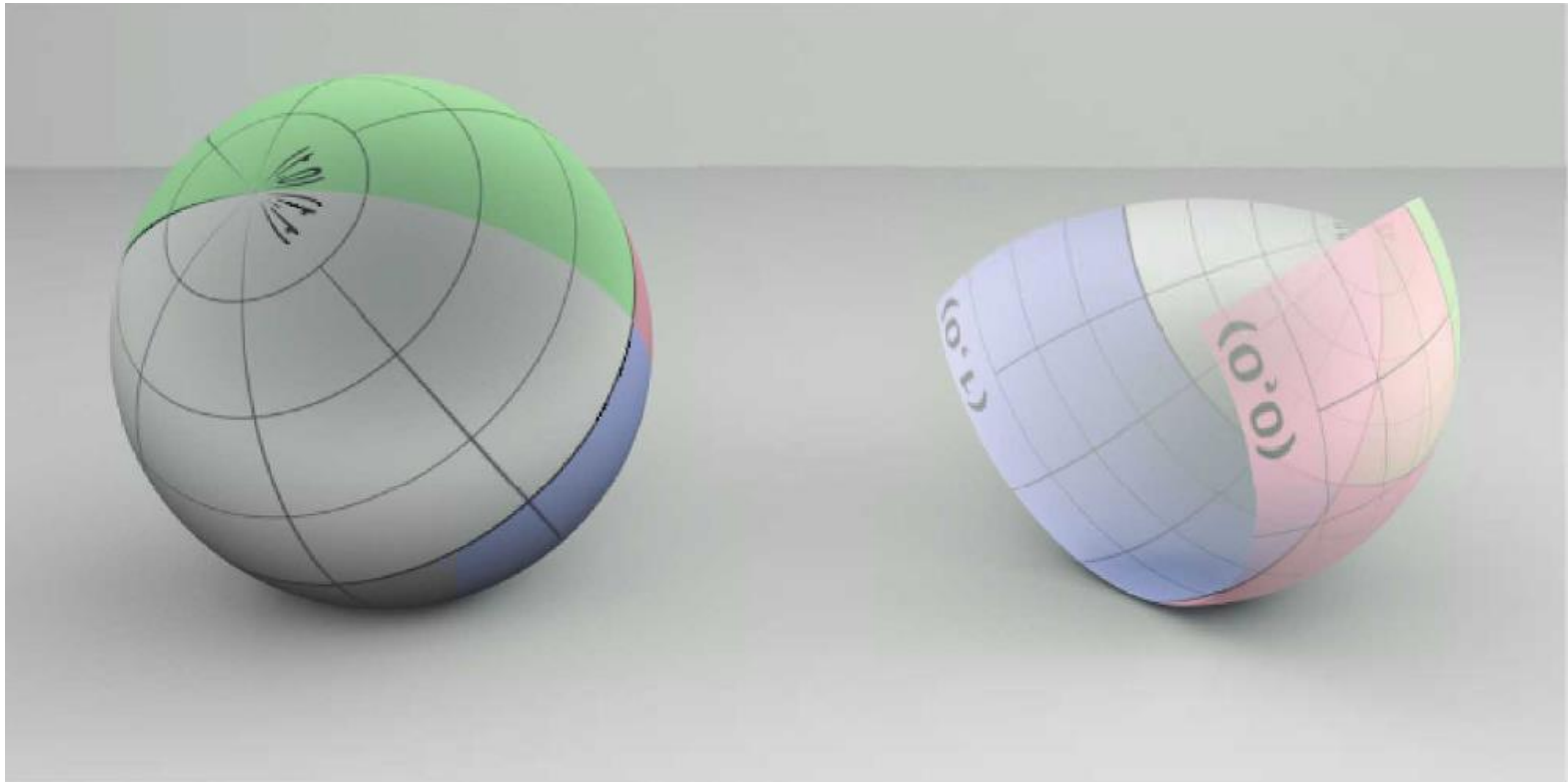
- **Cylinder**

- $p(u, v) = \text{Cylindrical2Cartesian}(r, 2 \pi u, H v)$ // cylinder height H



Parametric Surfaces

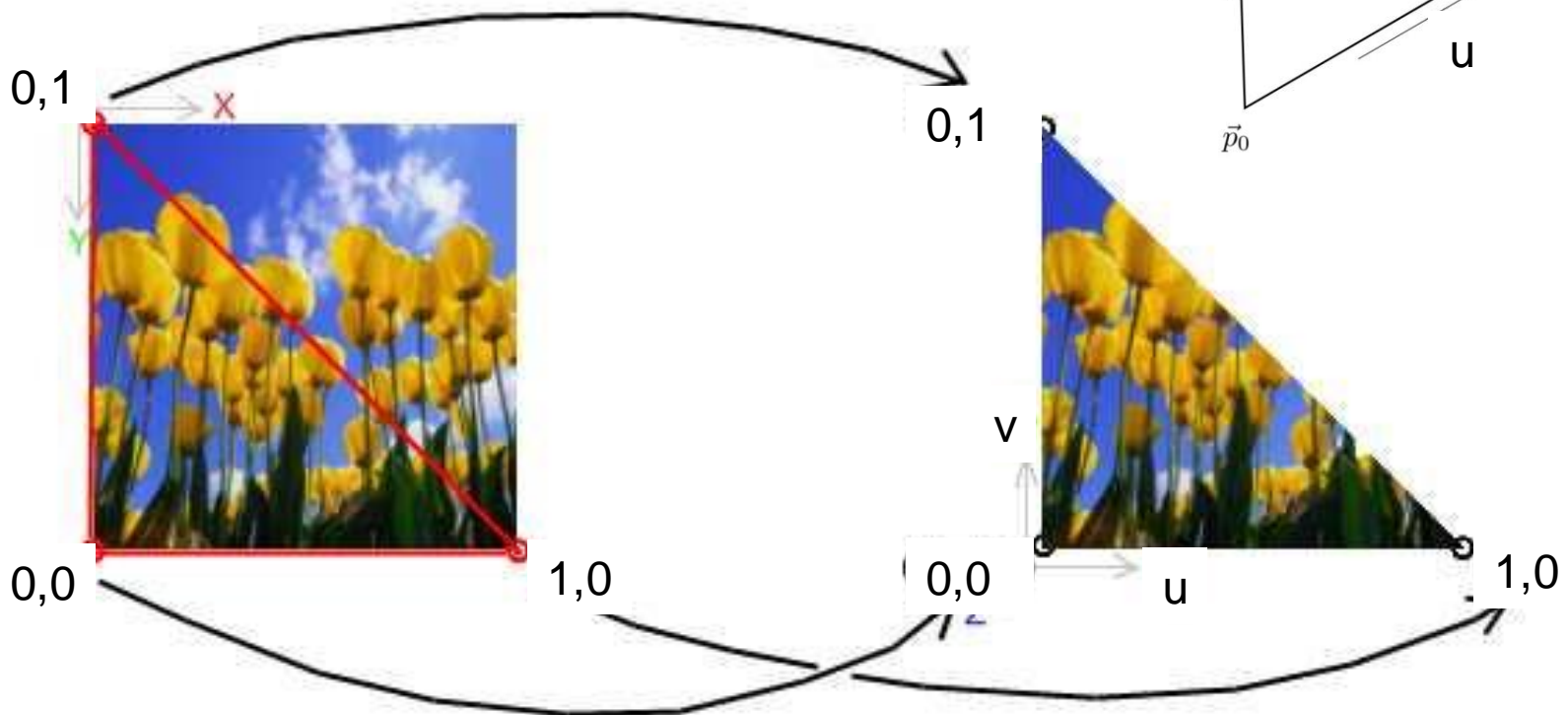
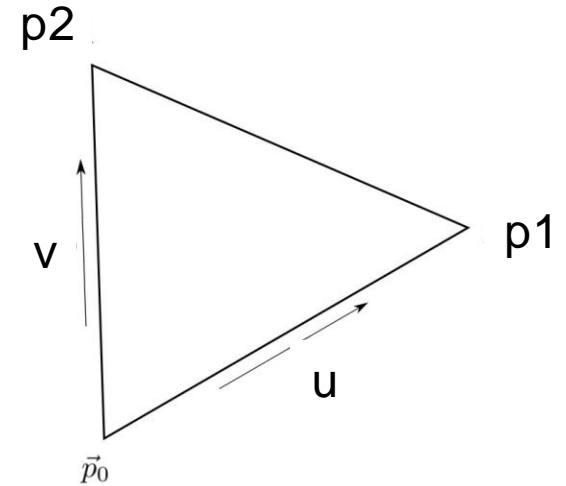
- **Spherical Coordinates**
 - $(x, y, z) = \text{Spherical2Cartesian}(r, \theta, \varphi)$
- **Sphere**
 - $p(u, v) = \text{Spherical2Cartesian}(r, \pi v, 2 \pi u)$



Parametric Surfaces

- **Triangle**

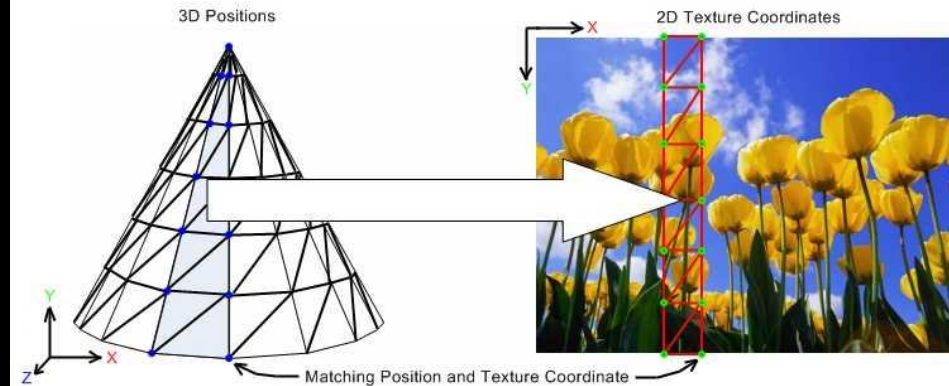
- Use barycentric coordinates directly
- $p(u, v) = (1 - u - v)p_0 + up_1 + vp_2$



Parametric Surfaces

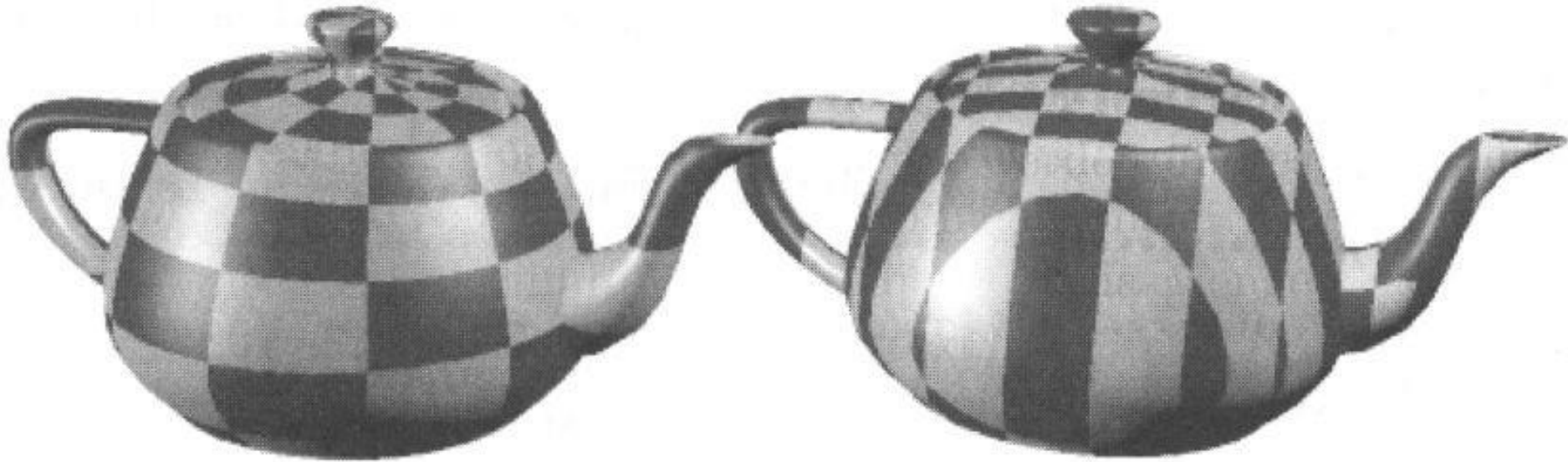
- **Triangle Mesh**

- Associate a predefined texture coordinate to each triangle vertex
 - Interpolate texture coordinates using barycentric coordinates
 - $u = \lambda_0 p_{0u} + \lambda_1 p_{1u} + \lambda_2 p_{2u}$
 - $v = \lambda_0 p_{0v} + \lambda_1 p_{1v} + \lambda_2 p_{2v}$
- Texture mapped onto manifold
 - Single texture shared by many triangles



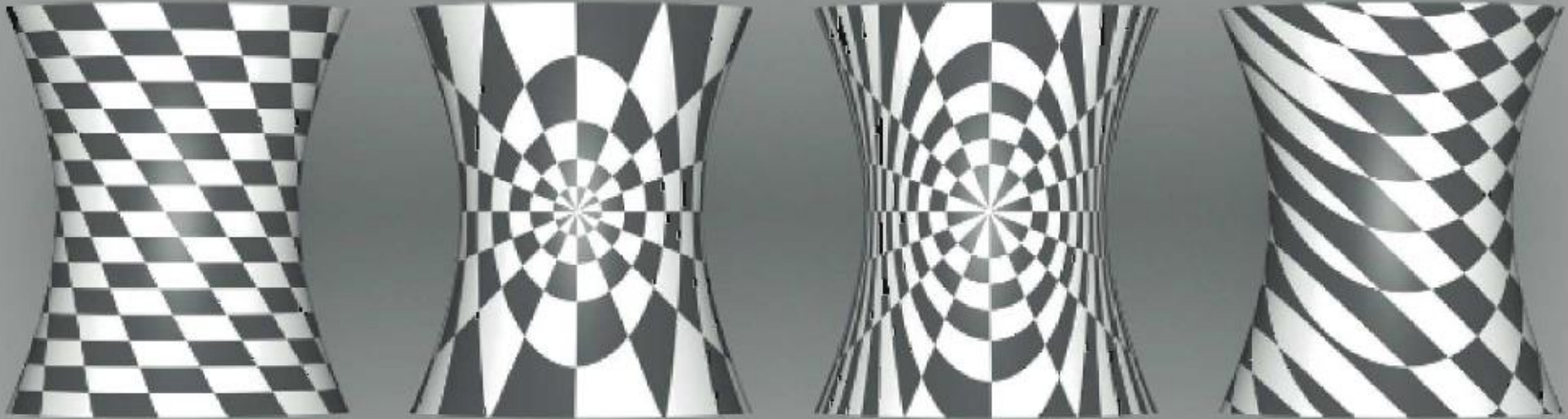
Surface Parameterization

- **Other Surfaces**
 - No intrinsic parameterization??



Intermediate Mapping

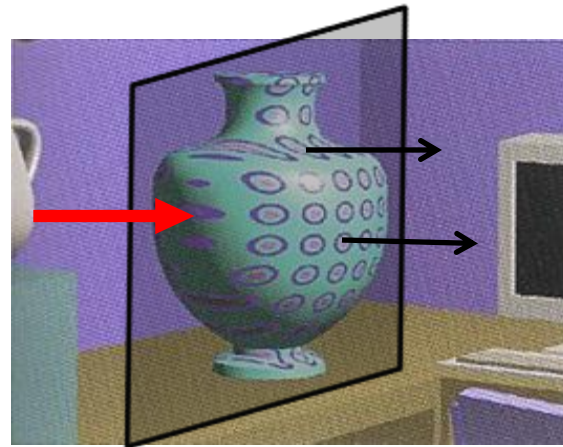
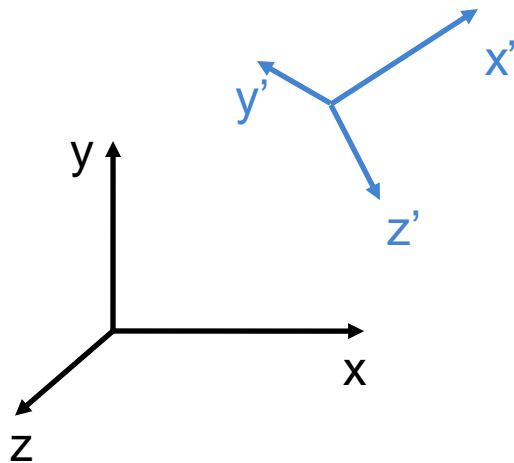
- **Coordinate System Transform**
 - Express Cartesian coordinates into a given coordinate system
- **3D to 2D Projection**
 - Drop one coordinate
 - Compute u and v from remaining 2 coordinates



Intermediate Mapping

- **Planar Mapping**

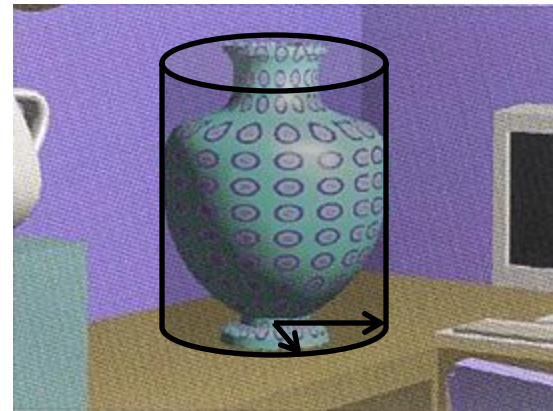
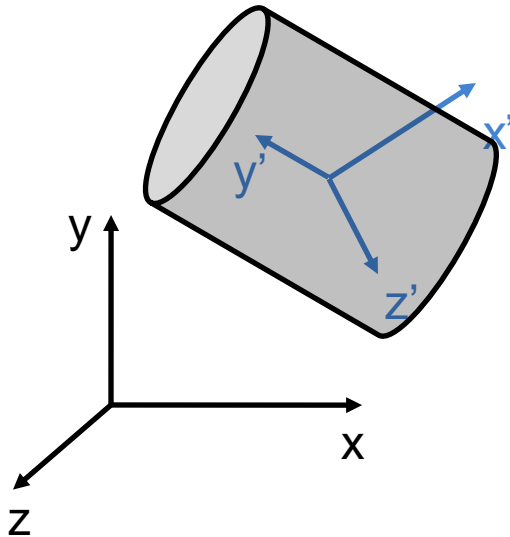
- Map to different Cartesian coordinate system
- $(x', y', z') = \text{AffineTransformation}(x, y, z)$
 - Orthogonal basis: translation + row-vector rotation matrix
 - Non-orthogonal basis: translation + inverse column-vector matrix
- Drop z' , map $u = x'$, map $v = y'$
- E.g.: Issues when surface normal orthogonal to projection axis



Intermediate Mapping

- **Cylindrical Mapping**

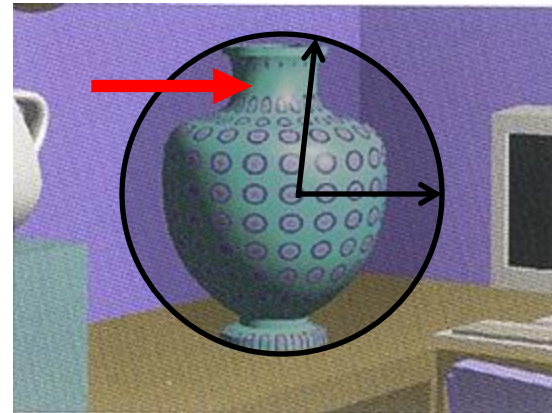
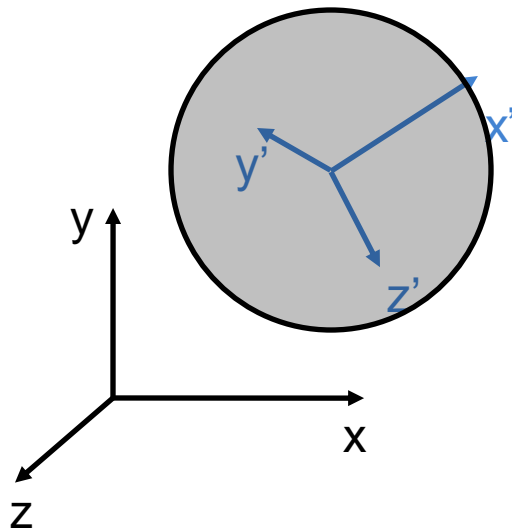
- Map to cylindrical coordinates (possibly after translation/rotation)
- $(r, \phi, z) = \text{Cartesian2Cylindrical}(x, y, z)$
- Drop r , map $u = \phi / 2\pi$, map $v = z / H$
- Extension: add scaling factors: $u = \alpha \phi / 2\pi$
- E.g.: Similar topology gives reasonable mapping



Intermediate Mapping

- **Spherical Mapping**

- Map to spherical coordinates (possibly after translation/rotation)
- $(r, \theta, \varphi) = \text{Cartesian2Spherical}(x, y, z)$
- Drop r , map $u = \varphi / 2\pi$, map $v = \theta / \pi$
- Extension: add scaling factors to both u and v
- E.g.: Issues in concave regions

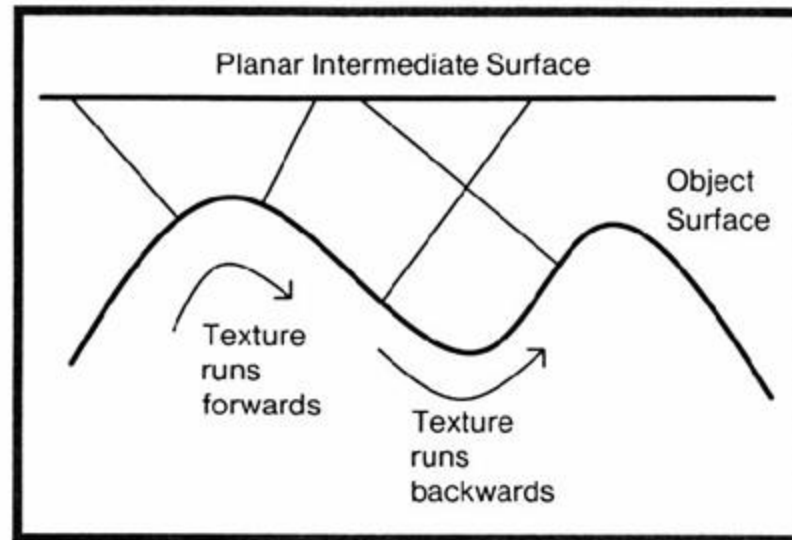


Two-Stage Mapping: Problems

- **Problems**

- May introduce undesired texture distortions if the intermediate surface differs too much from the destination surface
- Still often used in practice because of its simplicity

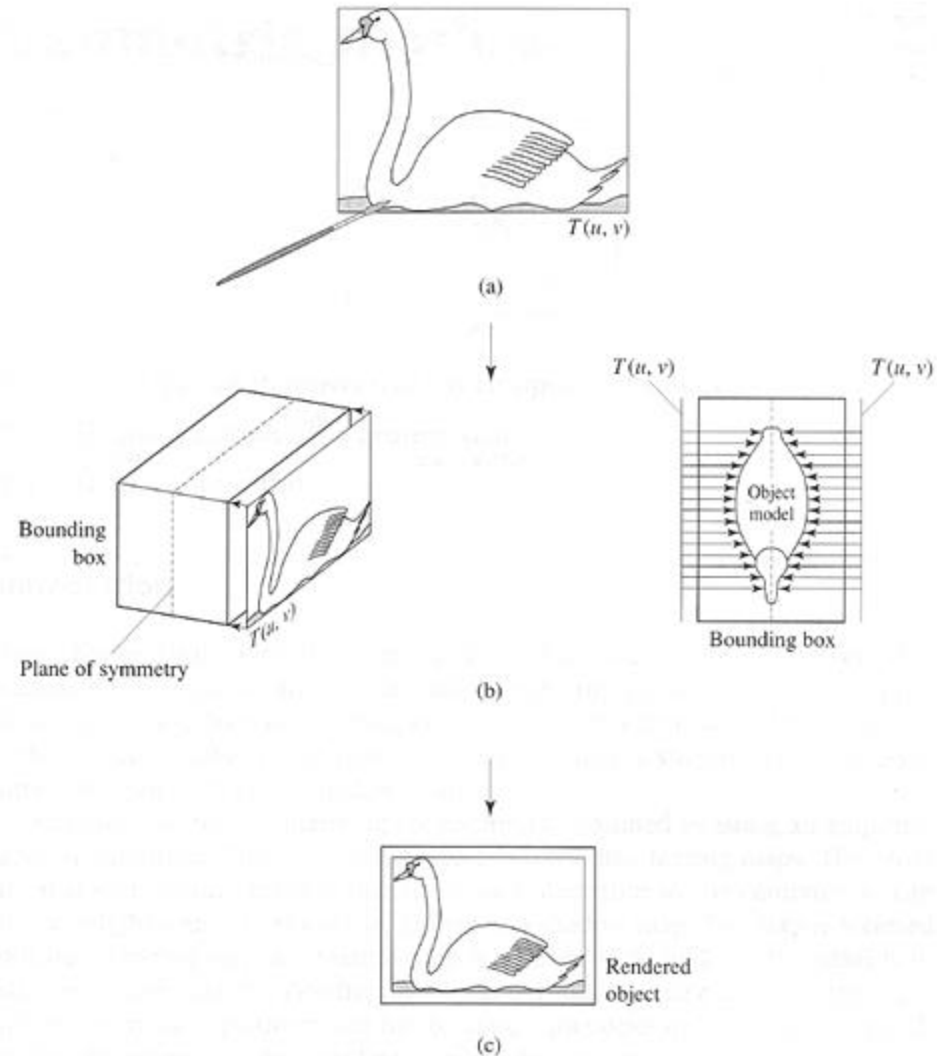
- **Example: Mapping point to plane along normal at the point:**



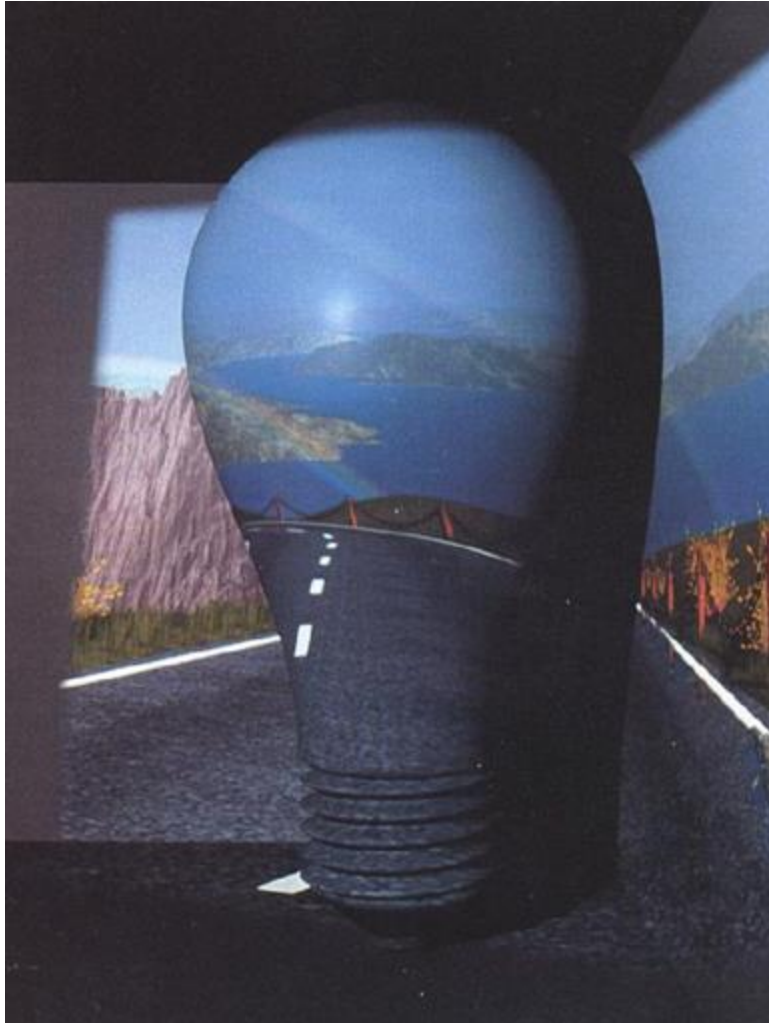
Surface concavities can cause the texture pattern to reverse if the object normal mapping is used.

Projective Textures

- **Project texture onto object surfaces**
 - Slide projector
- **Parallel or perspective projection**
- **Use photographs (or drawings) as textures**
 - Used a lot in film industry!
- **Multiple images**
 - View-dependent texturing (advanced topic)
- **Perspective Mapping**
 - Re-project photo on its 3D environment

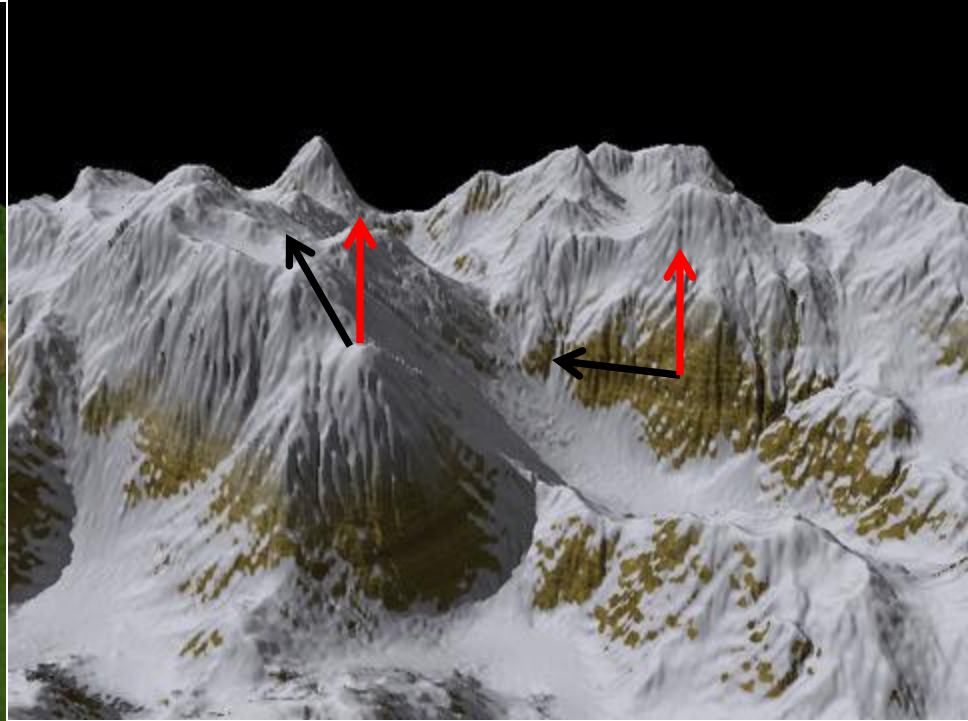
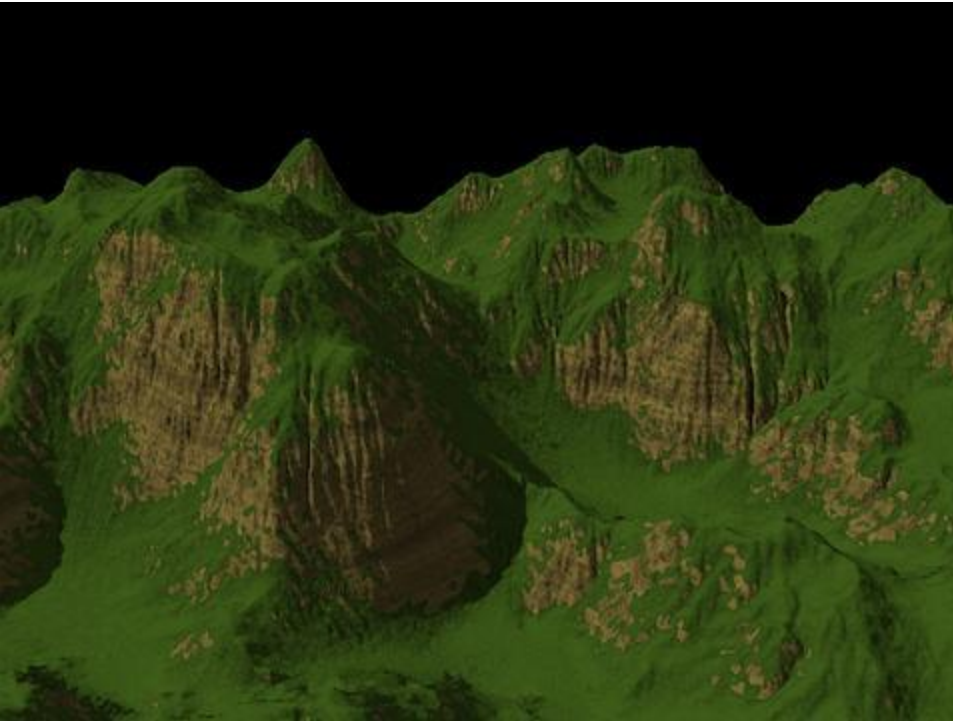


Projective Texturing: Examples



Slope-Based Mapping

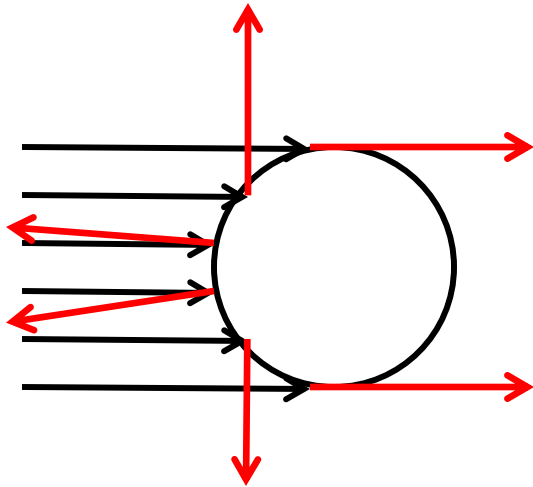
- **Definition**
 - Depends on surface normal and predefined vector
- **Example**
 - $\alpha = n \cdot \omega$
 - return α flatColor + (1 - α) slopeColor;



Environment Map

- **Spherical Map**

- Photo of a reflective sphere (gazing ball)
- Photos with a fish-eye camera
 - Only gives hemi-sphere mapping



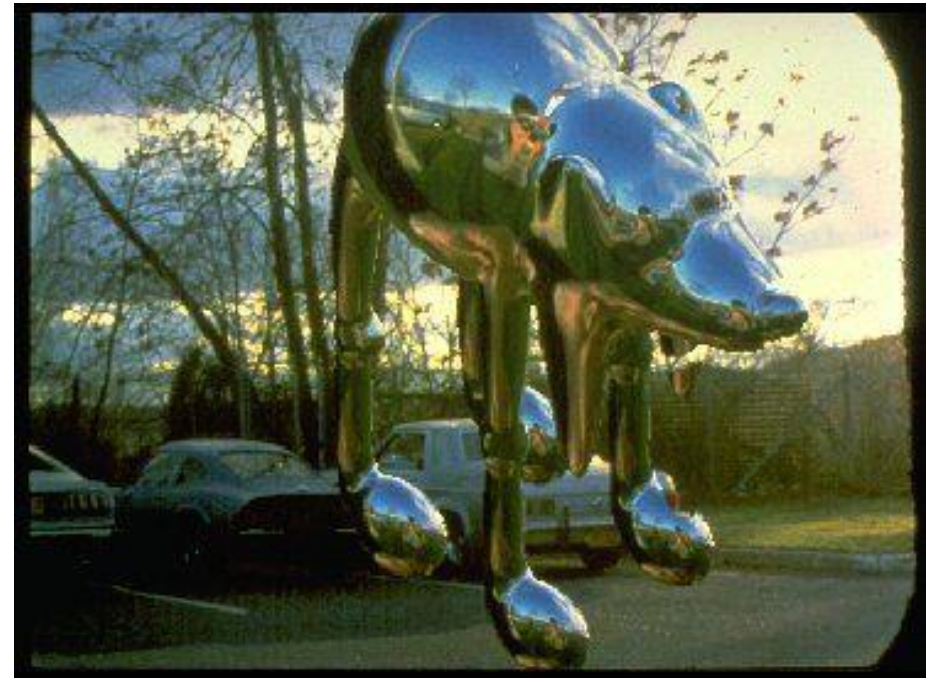
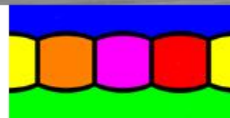
Environment Map

- **Latitude-Longitude Map**

- Remapping 2 images of reflective sphere
- Photo with an environment camera

- **Algorithm**

- If no intersection found, use ray direction to find background color
- Cartesian coords of ray dir. \rightarrow spherical coords \rightarrow uv tex coords



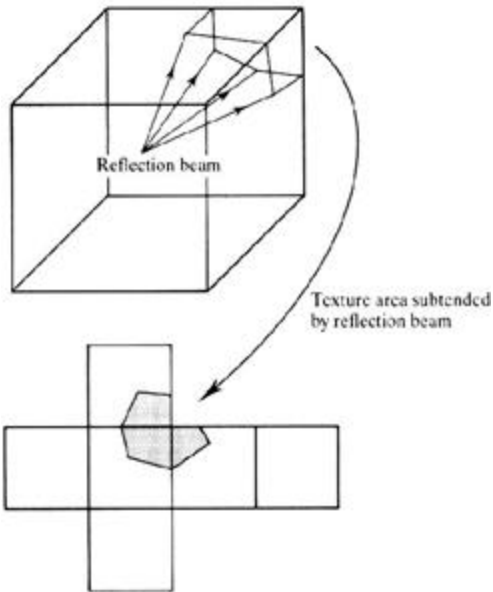
Environment Map

- **Cube Map**

- Remapping 2 images of reflective sphere
- Photos with a perspective camera

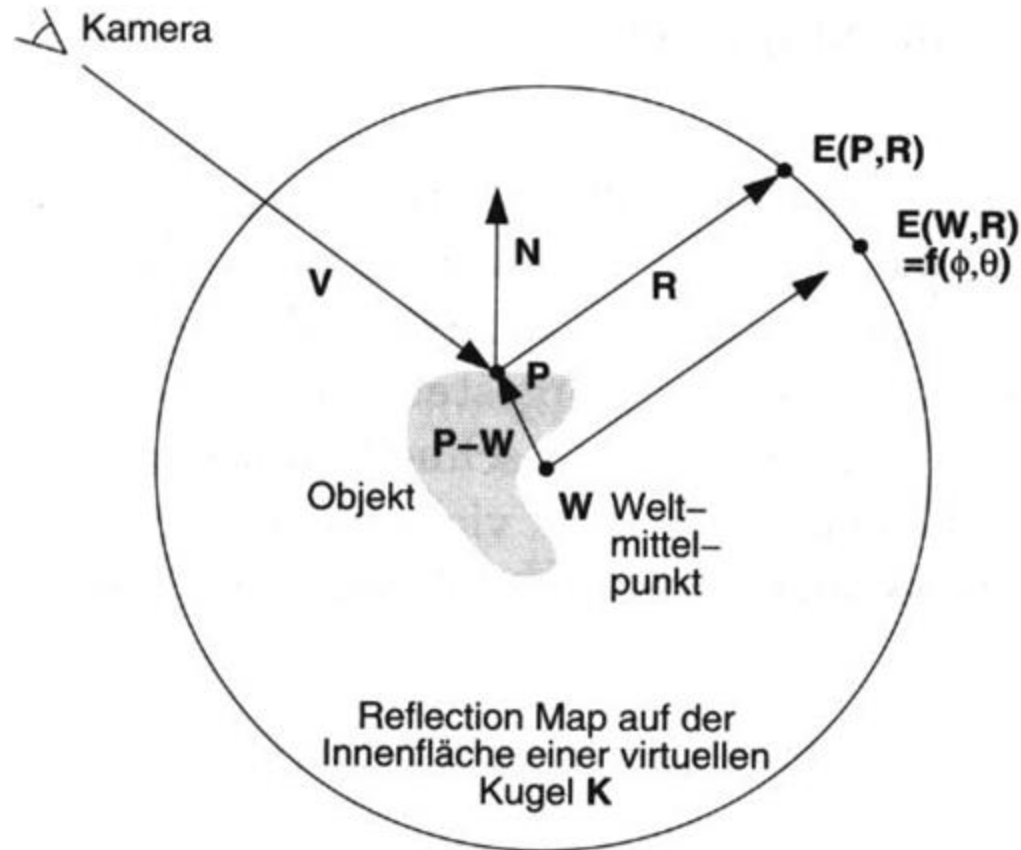
- **Algorithm**

- Find main axis (-x, +x, -y, +y, -z, +z) of ray direction
- Use other 2 coordinates to access corresponding face texture
 - Akin to a 90° projective light



Reflection Map Rendering

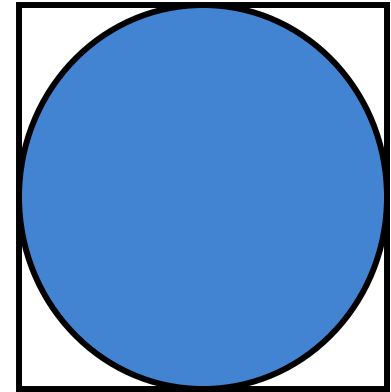
- Spherical parameterization
- O-mapping using reflected view ray intersection



Reflection Map Parameterization

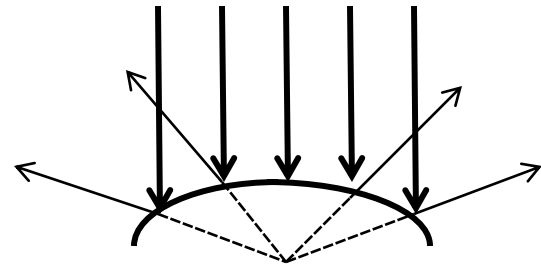
- **Spherical mapping**

- Single image
- Bad utilization of the image area
- Bad scanning on the edge
- Artifacts, if map and image do not have the same view point



- **Double parabolic mapping**

- Yields spherical parameterization
- Subdivide in 2 images (front-facing and back-facing sides)
- Less bias near the periphery
- Arbitrarily reusable
- Supported by OpenGL extensions



Reflection Mapping Example



Terminator II motion picture

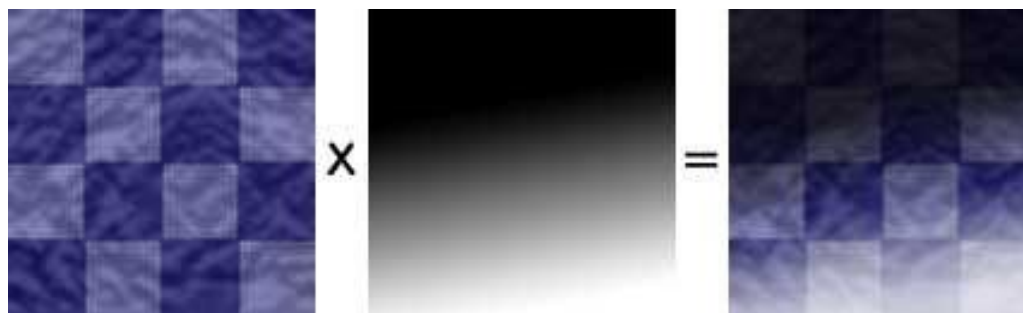
Reflection Mapping Example II

- **Reflection mapping with Phong reflection**
 - Two maps: diffuse & specular
 - Diffuse: index by surface normal
 - Specular: indexed by reflected view vector



Light Maps

- **Light maps (e.g. in Quake)**
 - Pre-calculated illumination (local irradiance)
 - Often very low resolution: smoothly varying
 - Multiplication of irradiance with base texture
 - Diffuse reflectance only
 - Provides surface radiosity
 - View-independent out-going radiance
 - Animated light maps
 - Animated shadows, moving light spots, etc...

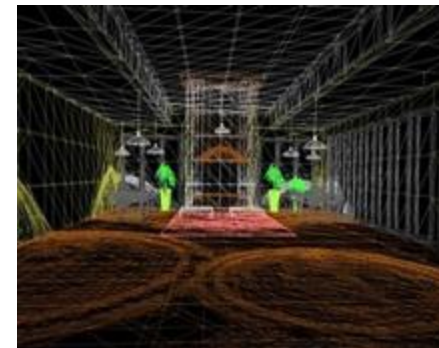


Reflectance

Irradiance

Radiosity

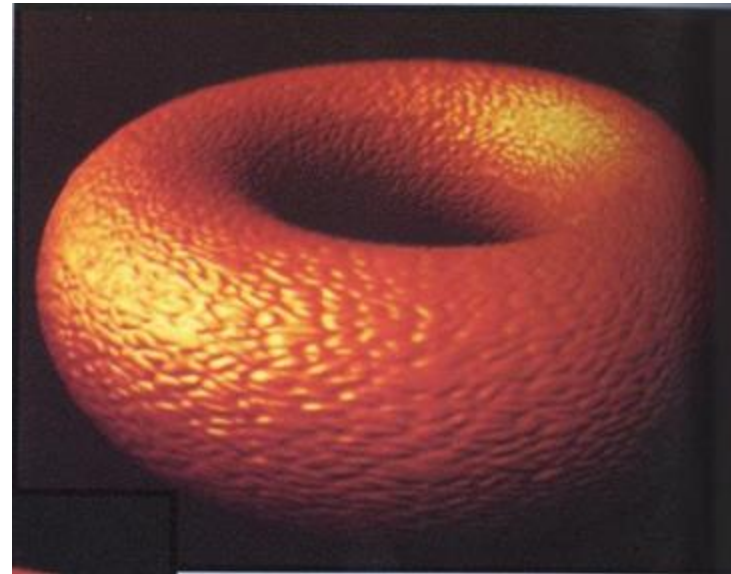
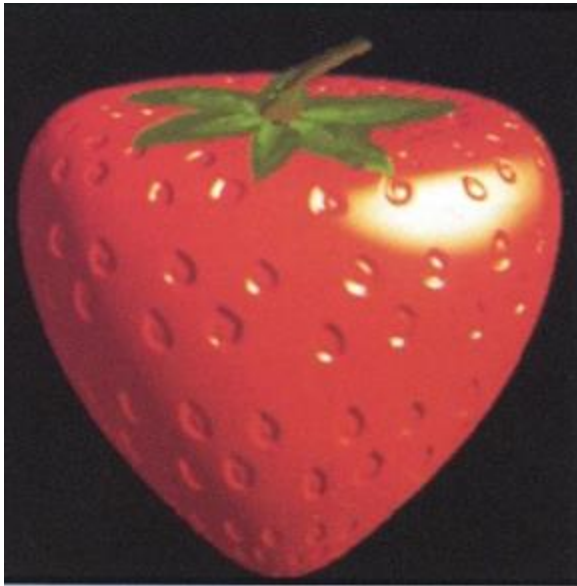
$$B(x) = \rho(x) E(x) = \pi L_o(x)$$



Representing radiosity
in a mesh or texture

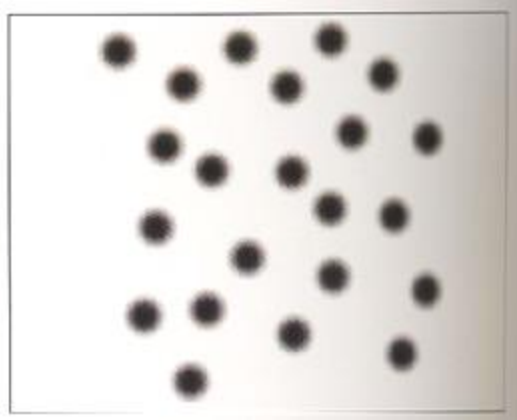
Bump Mapping

- **Modulation of the normal vector**
 - Surface normals changed only
 - Influences shading only
 - No self-shadowing, contour is **not** altered

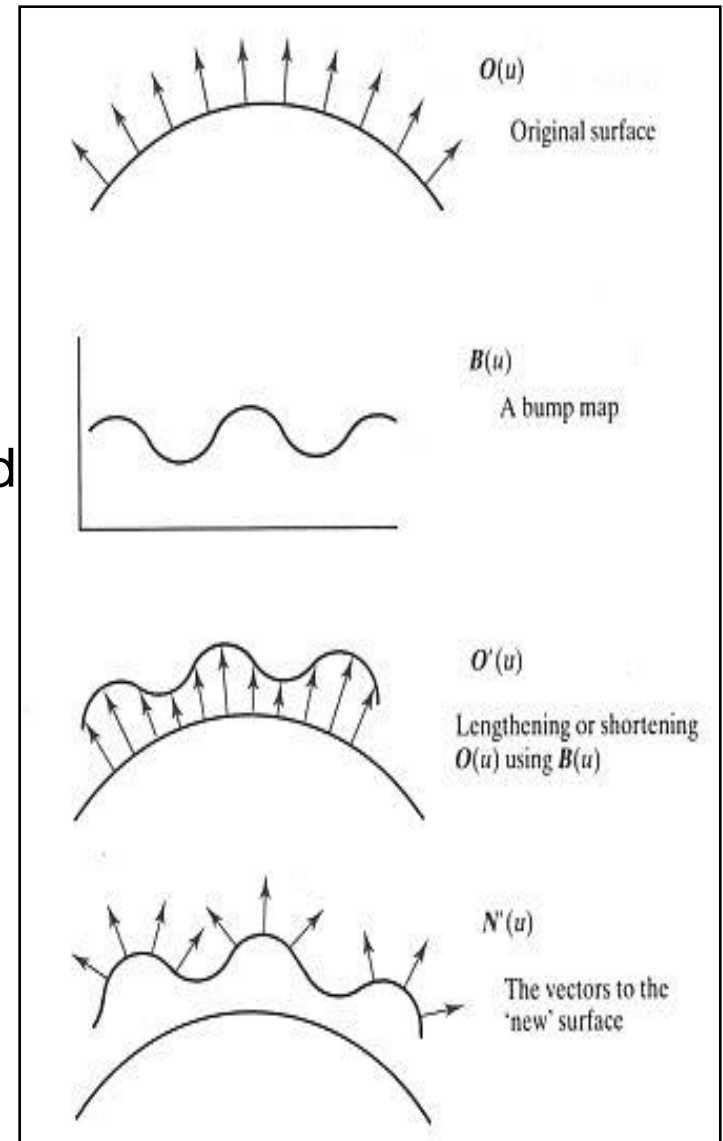


Bump Mapping

- **Original surface:** $O(u, v)$
 - Surface normals are known
- **Bump map:** $B(u, v) \in R$
 - Surface is offset in normal direction according to bump map intensity
 - New normal directions $N'(u, v)$ are calculated based on virtually displaced surface $O'(u, v)$
 - Original surface is rendered with new normals $N'(u, v)$



Grey-valued texture used for bump height



Bump Mapping

$$O'(u, v) = O(u, v) + B(u, v) \frac{N}{|N|}$$

- Normal is cross-product of derivatives:

$$O'_u = O_u + B_u \frac{N}{|N|} + B \left(\frac{N}{|N|} \right)_u$$

$$O'_v = O_v + B_v \frac{N}{|N|} + B \left(\frac{N}{|N|} \right)_v$$

- If B is small (i.e., the bump map displacement function is small compared to its spatial extent) the last term in each equation can be ignored

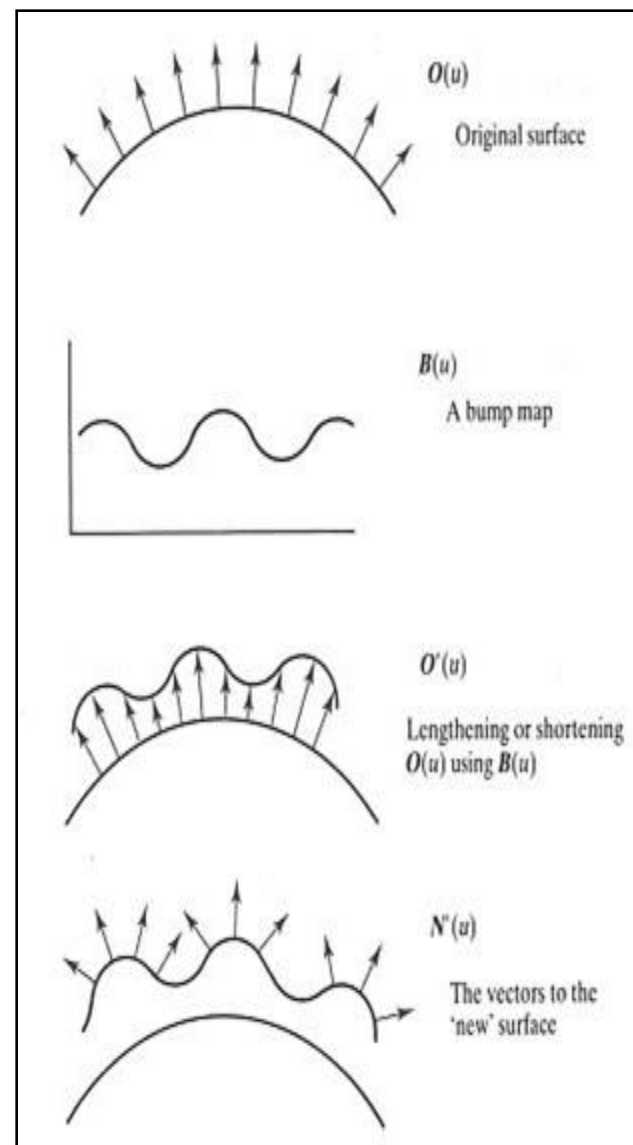
$$N'(u, v)$$

$$= O_u \times O_v + B_u \left(\frac{N}{|N|} \times O_v \right) + B_v \left(O_u \times \frac{N}{|N|} \right) + B_u B_v \left(\frac{N \times N}{|N|^2} \right)$$

- The first term is the normal to the surface and the last is zero, giving:

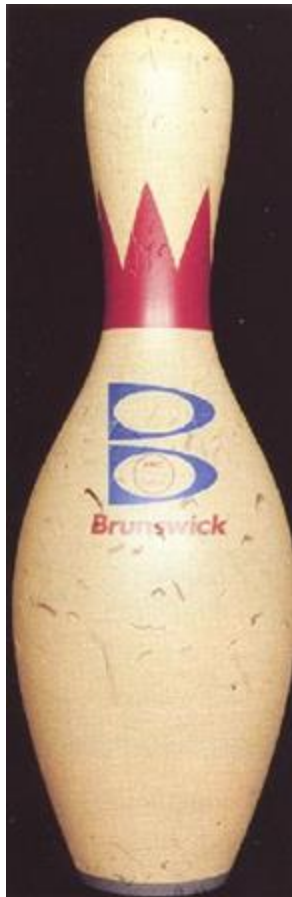
$$D = B_u(N \times O_v) - B_v(N \times O_u)$$

$$N' = N + D$$

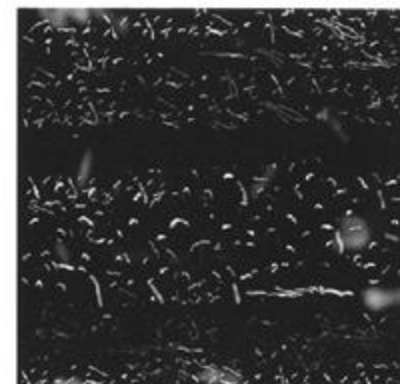


Texture Examples

- **Complex optical effects**
 - Combination of multiple texture effects

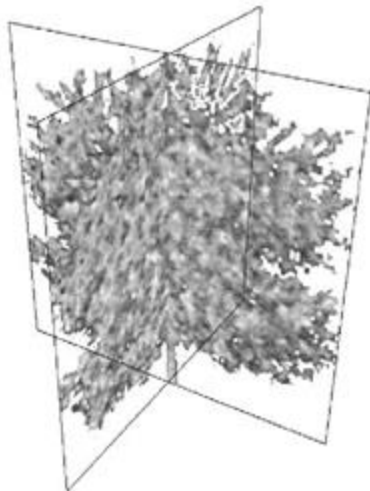
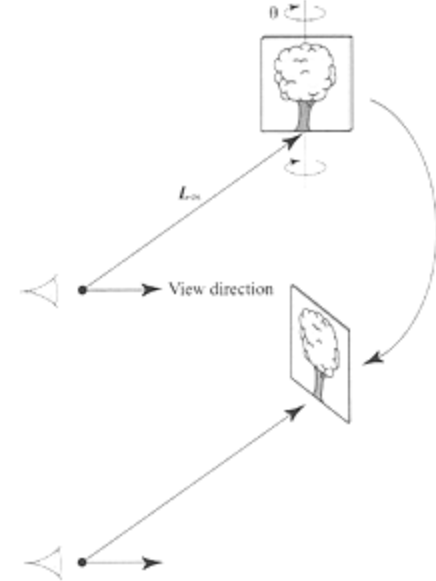


RenderMan Companion



Billboards / Transparency Map

- **Single textured polygons**
 - Often with opacity texture
 - Rotates, always facing viewer
 - Used for rendering distant objects
 - Best results if approximately radially or spherically symmetric
- **Multiple textured polygons**
 - Azimuthal orientation: different orientations
 - Complex distribution: trunk, branches, ...



Opacity texture

