Computer Graphics

- Material Models -

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(Some slides from Prof. Wenzel Jakob)

Overview

- **Reflectance Properties**
- **Bidirectional Reflectance Distribution Function**
- **BRDF Models**

REFLECTANCE PROPERTIES

Conductor vs. Dielectric

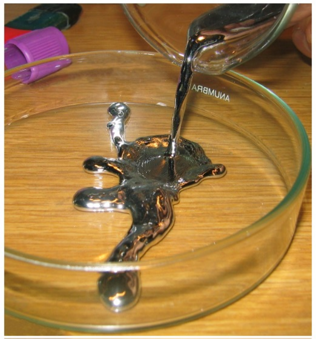
Copper

Iron

Glass

Ethanol

Gold



Mercury

Water

Air

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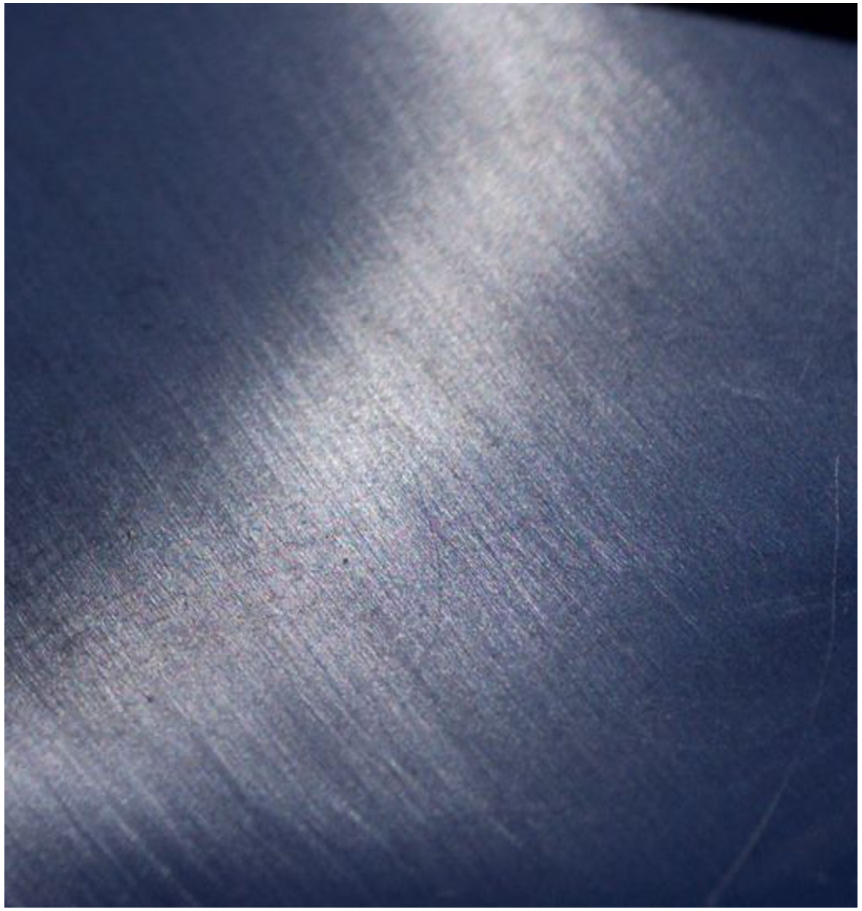
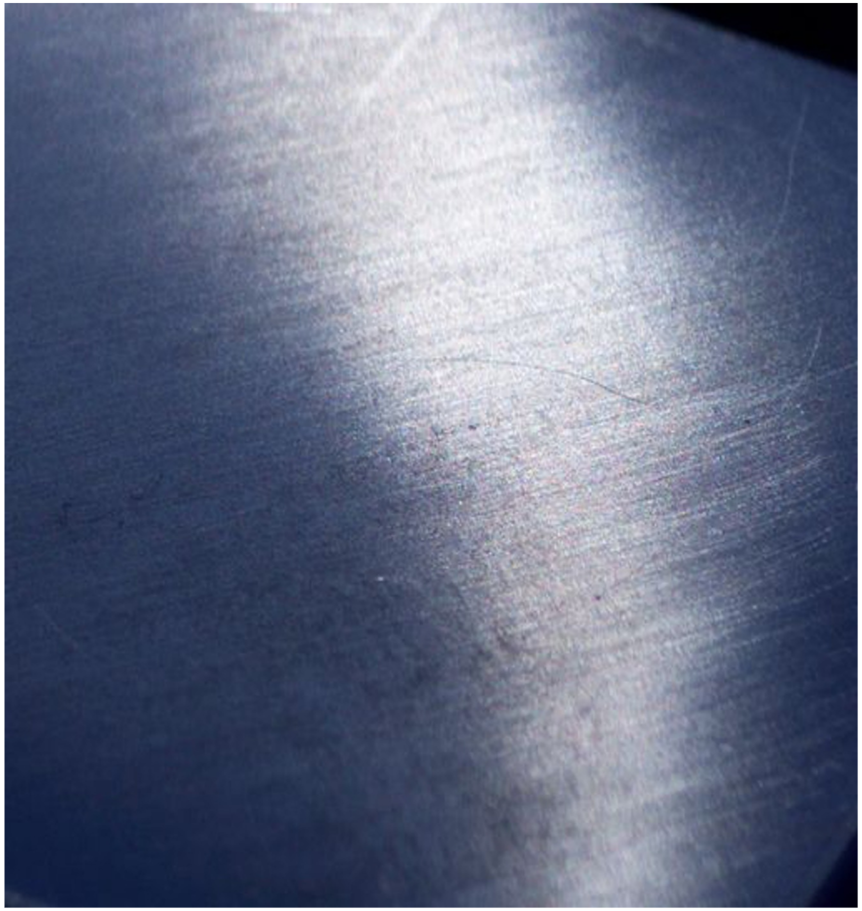
Anisotropy





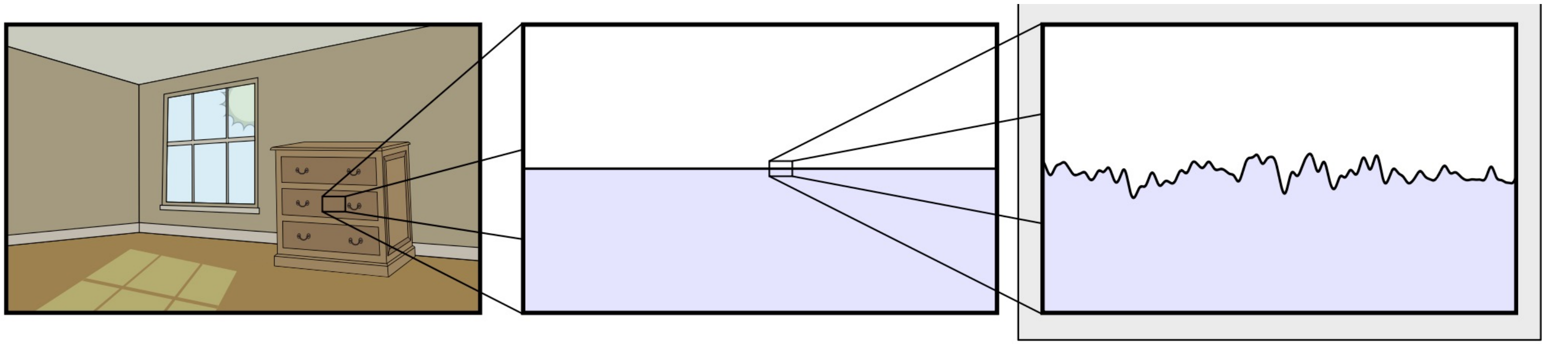
Irridescence

Fibers

Three different levels of detail

- **Key idea**
  - transition from individual interactions to statistical averages



Macro scale

Scene geometry

Meso scale

Detail at intermediate scales

(can have variations here too)

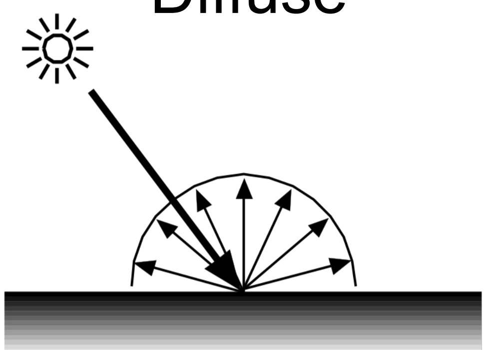
Micro scale

Roughness

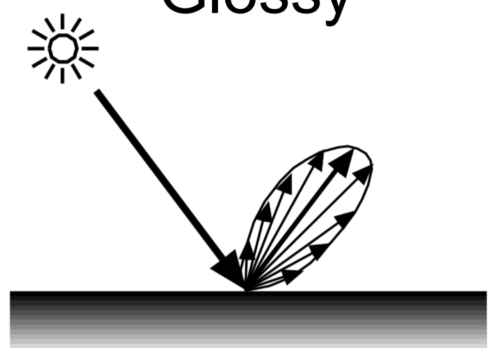
Microscopic Roughness

- **Light source at exactly the same position**

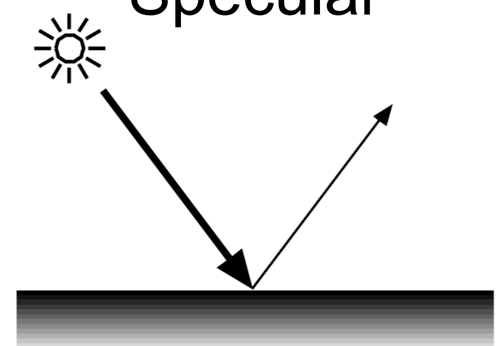
Diffuse



Glossy

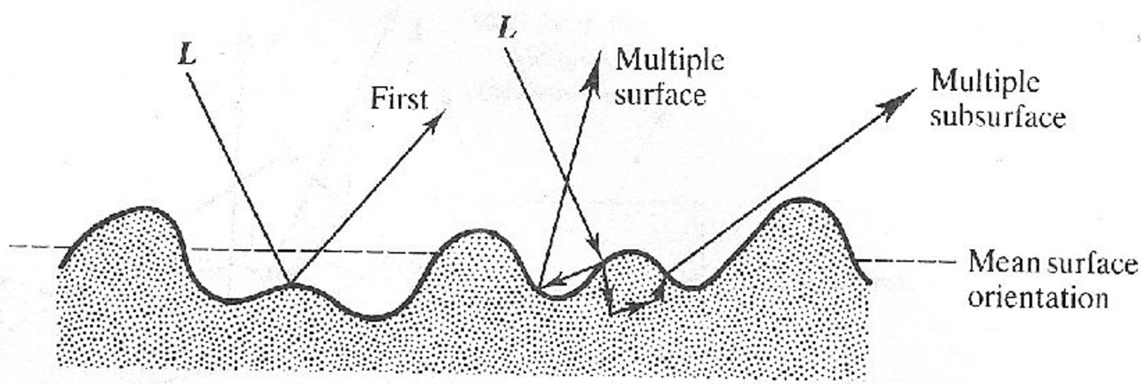


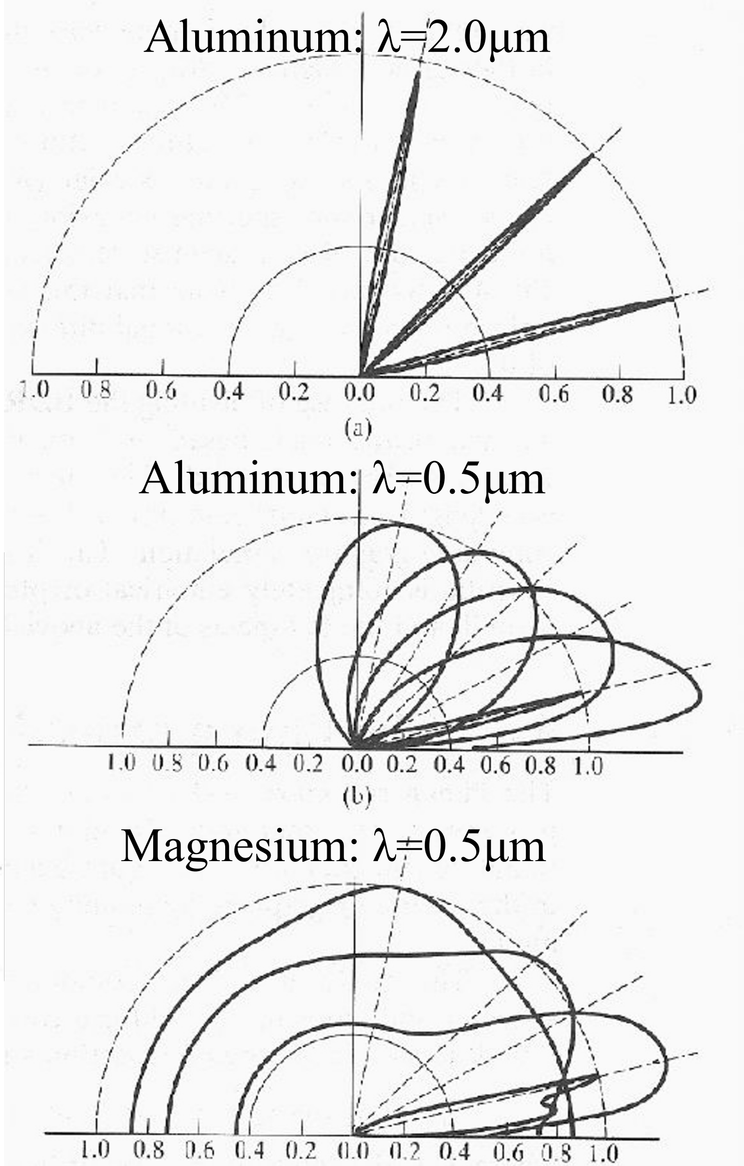
Specular



Reflectance

- **Depends on**
  - Wavelength
  - Absorption
  - Surface micro-geometry
  - Micro-scale scattering
  - Index of refraction / dielectric constant

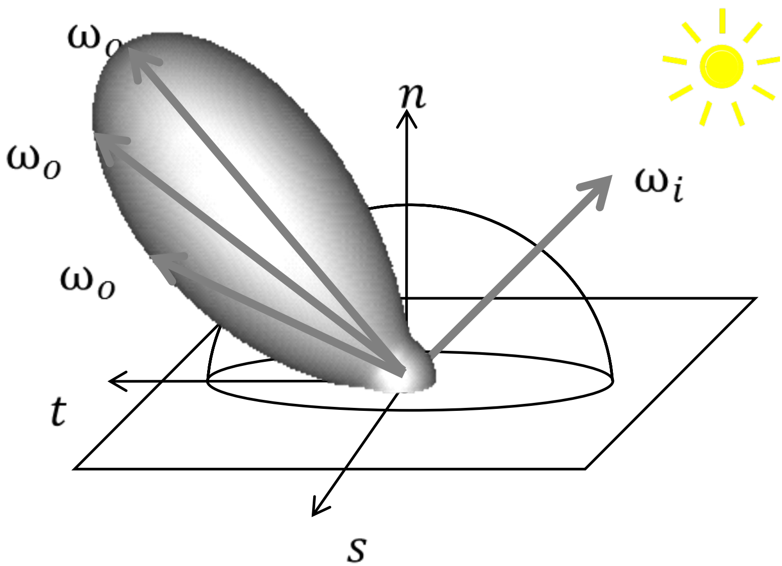




BRDF

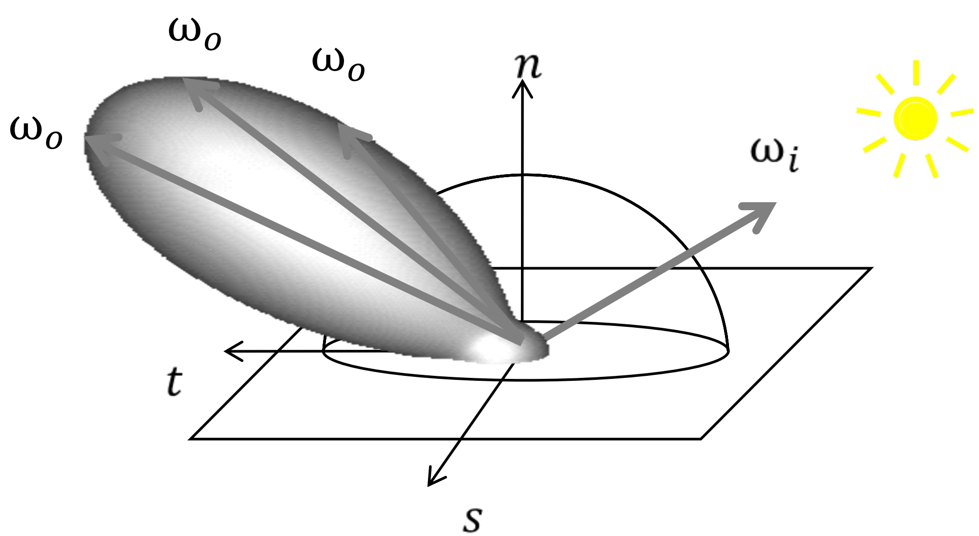
Representation

- **Statistical Distribution**
  - Aggregate distribution of light reflected off an opaque surface



Representation

- **Bidirectional**
  - Depends on both view and light directions



Definition

- **Derivative of Reflected Radiance**

$$dL_r(x,\omega_o) = f_r(x,\omega_i,\omega_o)L_i(x,\omega_i)\cos\theta_i \, d\omega_i$$

- **BRDF**
  - Ratio of reflected radiance to incident irradiance

$$f_r(x,\omega_i,\omega_o) = \frac{dL_r(x,\omega_o)}{L_i(x,\omega_i)\cos\theta_i d\omega_i} = \frac{dL_r(x,\omega_o)}{dE_i(x,\omega_i)}$$

Properties

- **Units**
  - Inverse steradian: sr^{-1}
- **Values**
  - Distribution function
    - Positive
    - Not necessarily finite
  - Range
    - From 0: absorption
    - To ∞ : reflection, δ-function

Properties

- **Energy Conservation**
  - Cannot reflect more light than incident amount
  - Can reflect less light than incident amount: absorption

$$\int_{\Omega} f_r(x, \omega_i, \omega_o) \cos\theta_o \, d\omega_o \leq 1 \quad \forall \omega_i$$

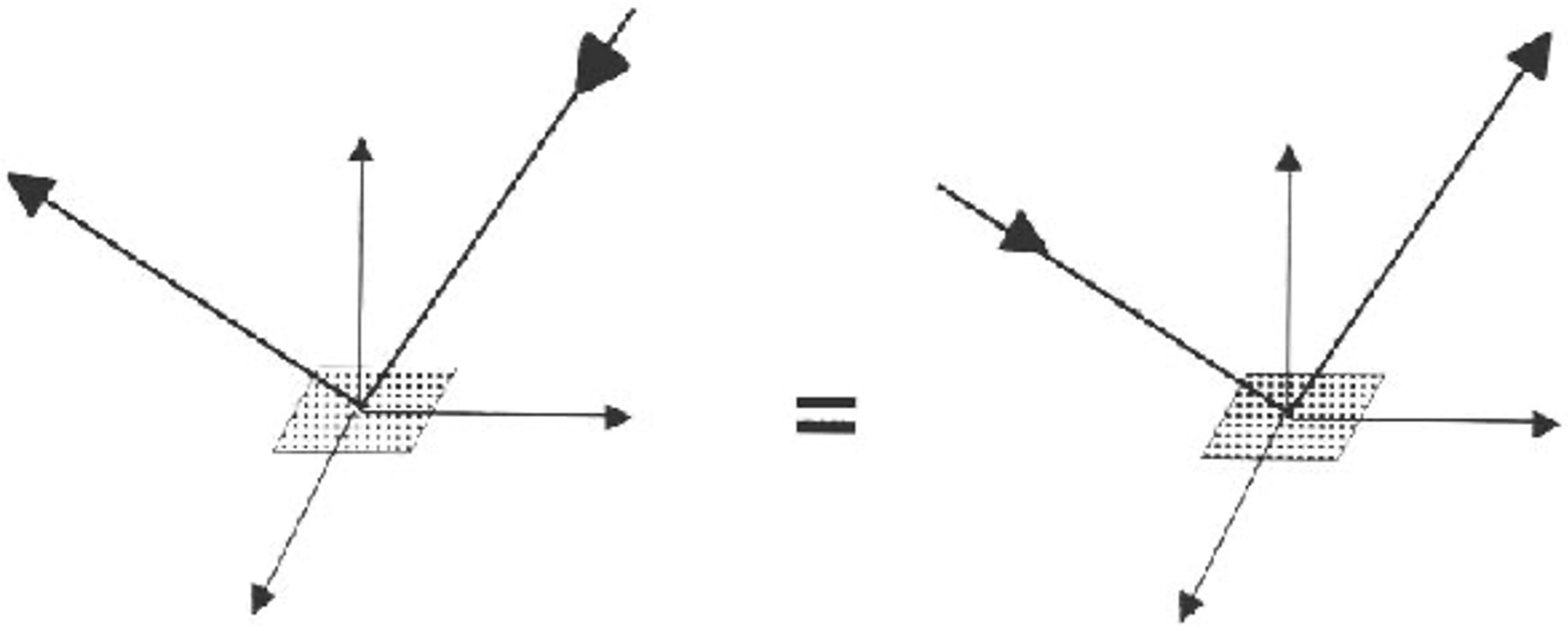
- **Albedo**
  - Directional-hemispherical reflectance
  - Reflected radiance in furnace
  - Fraction of incident flux density
    - Incoming from given direction
    - That is reflected in all directions
  - Dimensionless number in [0,1]
  - Depends on single direction

$$\rho(x, w) = \int_{\Omega} f_r(x, w, w_o) \cos\theta_o d\omega_o$$

Properties

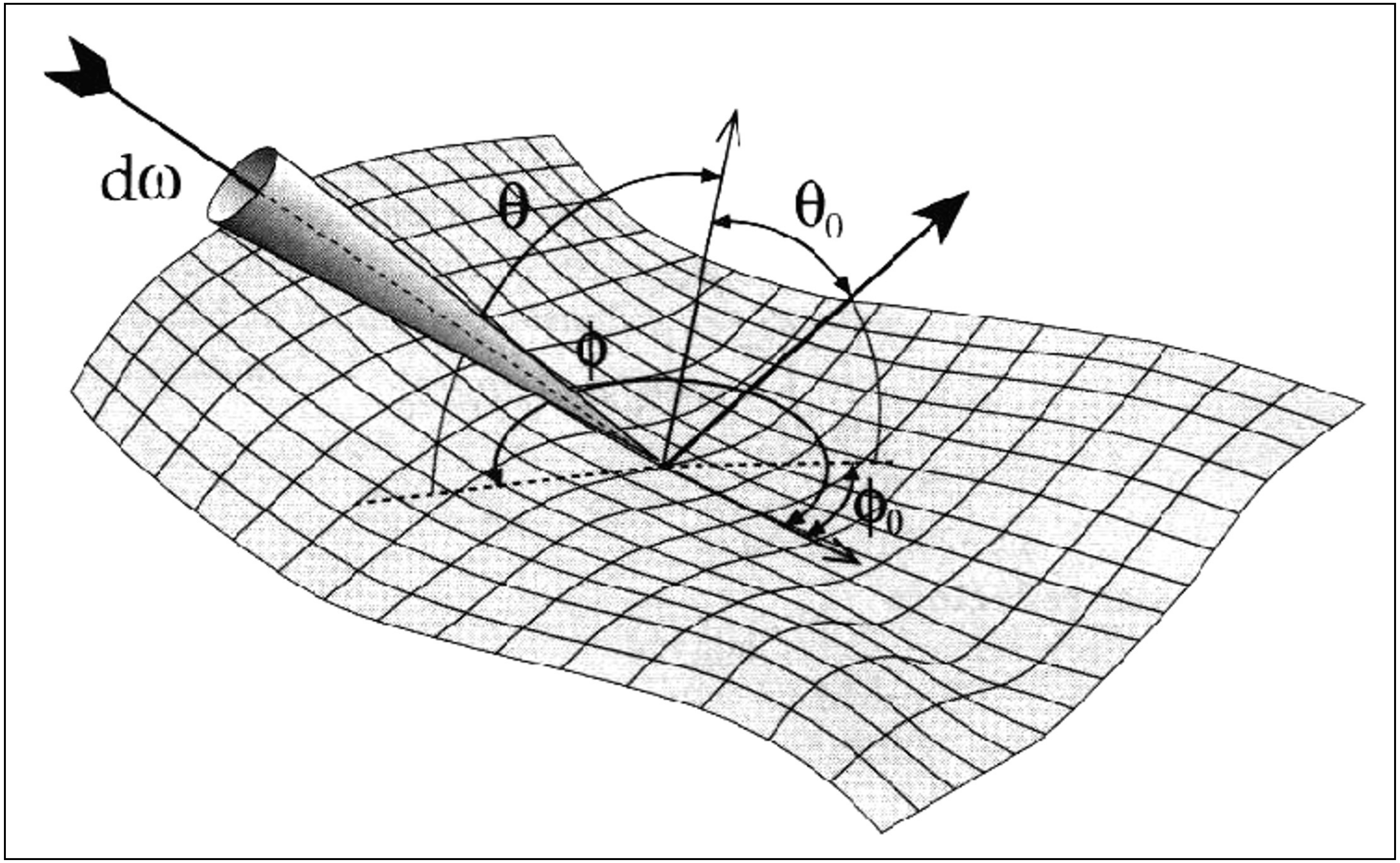
- **Helmholtz Reciprocity**
  - Swapping incident and reflected directions preserves reflectance

$$f_r(x, \omega_i, \omega_o) = f_r(x, \omega_o, \omega_i)$$



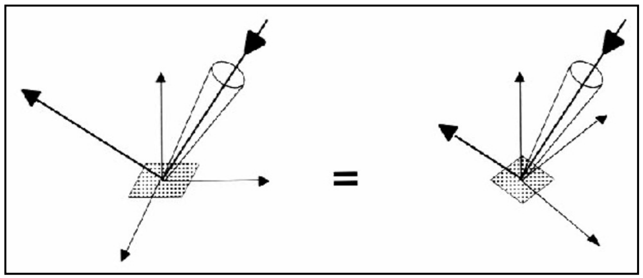
Parameterization

- **Describes Surface Reflection**
  - For light incident from direction *(θ_i, φ_i)*
  - Observed from direction *(θ_o, φ_o)*



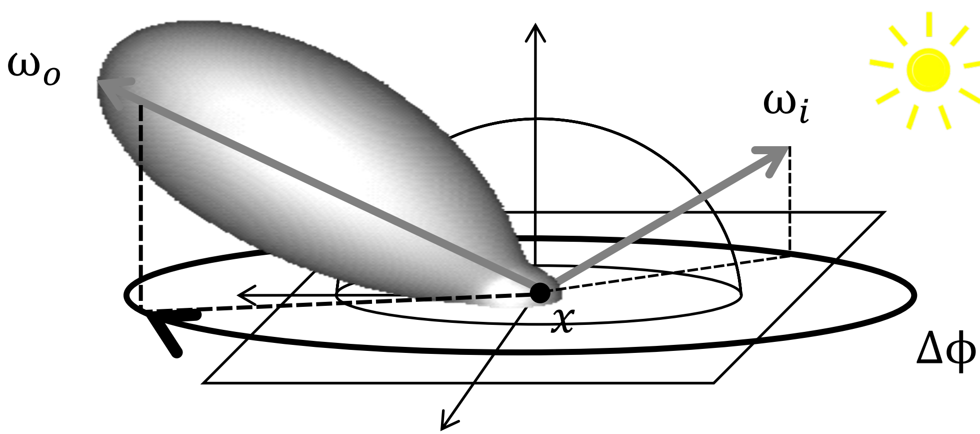
Isotropic BRDF – 3D

- **Invariant with respect to rotation about the normal**
  - Only depends on azimuth difference to incoming angle
  - Only 3 instead of 4 directional degrees of freedom



$$f_r(\Delta\phi, \theta_i \rightarrow \theta_o)$$

$$\Delta\phi = \phi_o - \phi_i$$

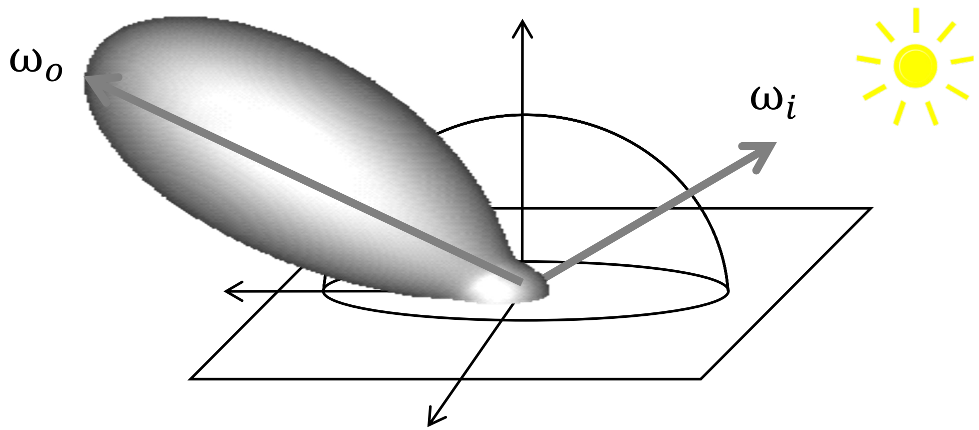


Homogeneous BRDF – 4D

- **Bidirectional Reflectance Distribution Function**

$$f_r(\omega_i \rightarrow \omega_o)$$

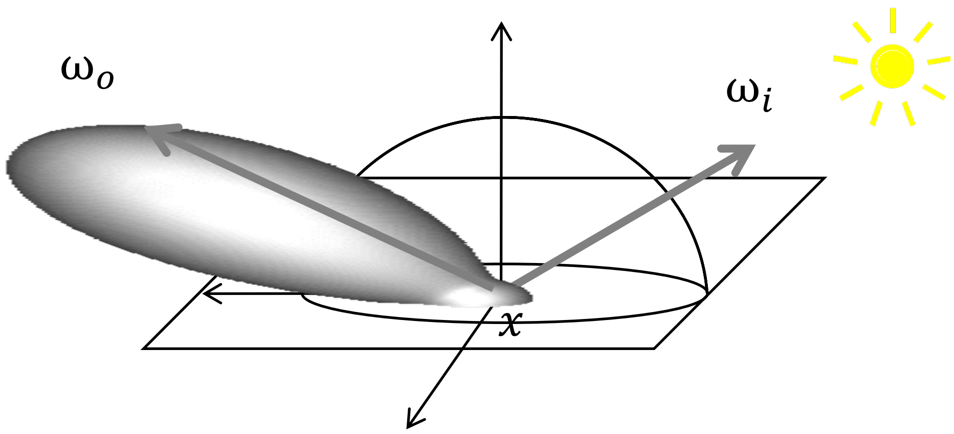
$$f_r\big((\theta_i, \phi_i) \rightarrow (\theta_o, \phi_o)\big)$$



Spatially-Varying BRDF – 6D

- **Heterogeneous Materials**
  - Dependent on position
  - Reflection at point of incidence: xi = xo

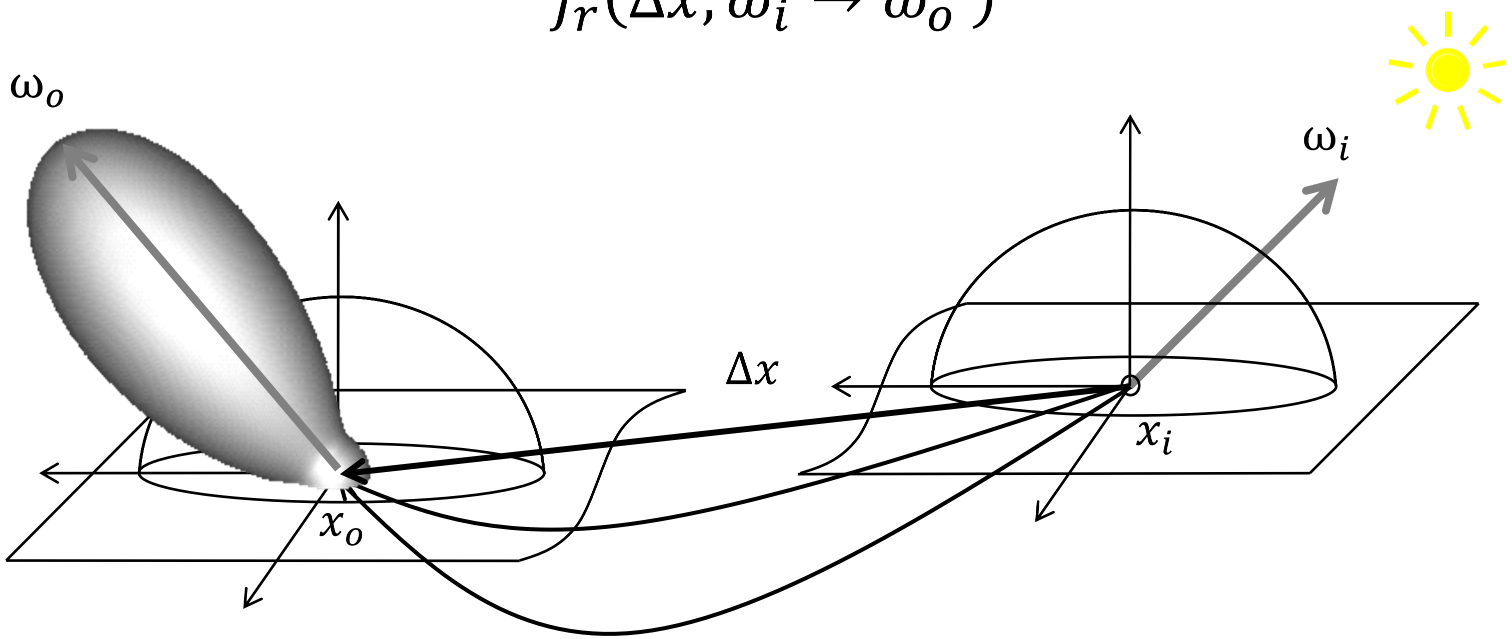
$$f_r(x, \omega_i \rightarrow \omega_o)$$



Homogeneous BSSRDF – 6D

- **Bidirectional Scattering Surface Reflectance Distribution Function**
  - Assumes a homogeneous and flat surface
  - Only depends on the difference to the outgoing point
  - Subsurface scattering

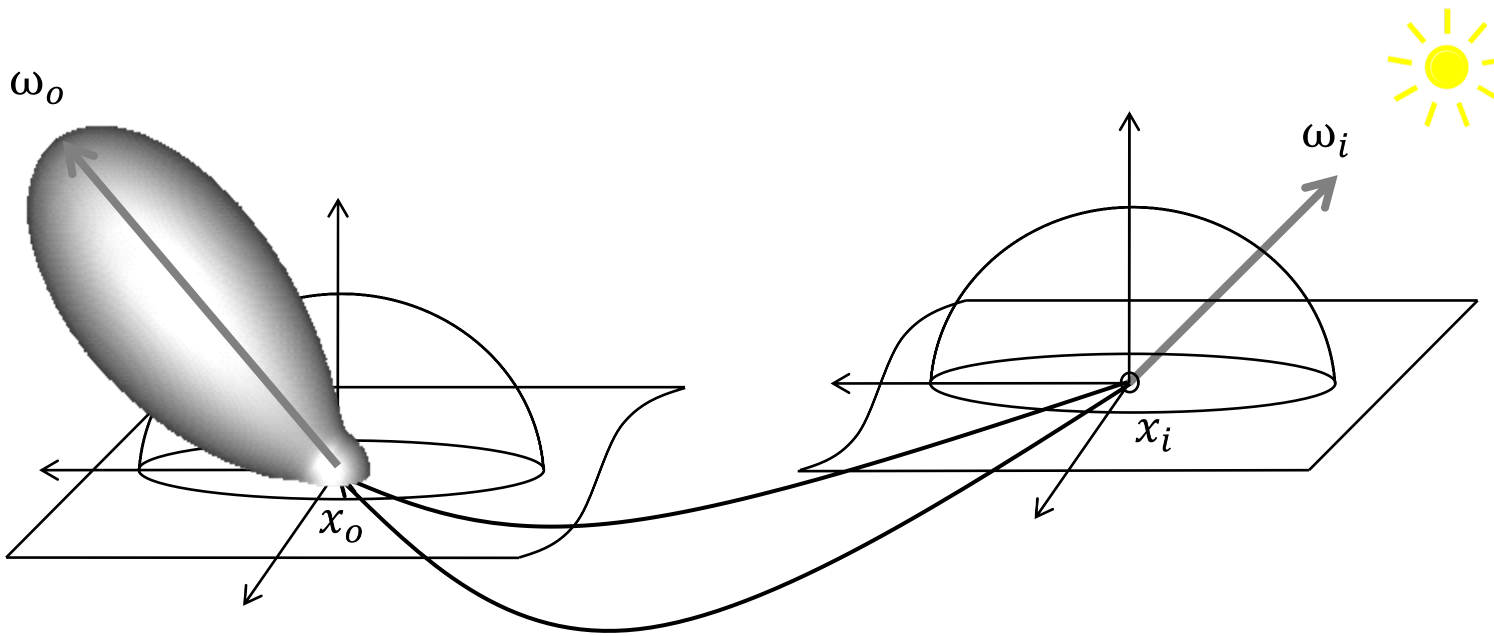
$$f_r(\Delta x, \omega_i \to \omega_o)$$



BSSRDF – 8D

- **Heterogeneous Materials**
  - Dependent on positions

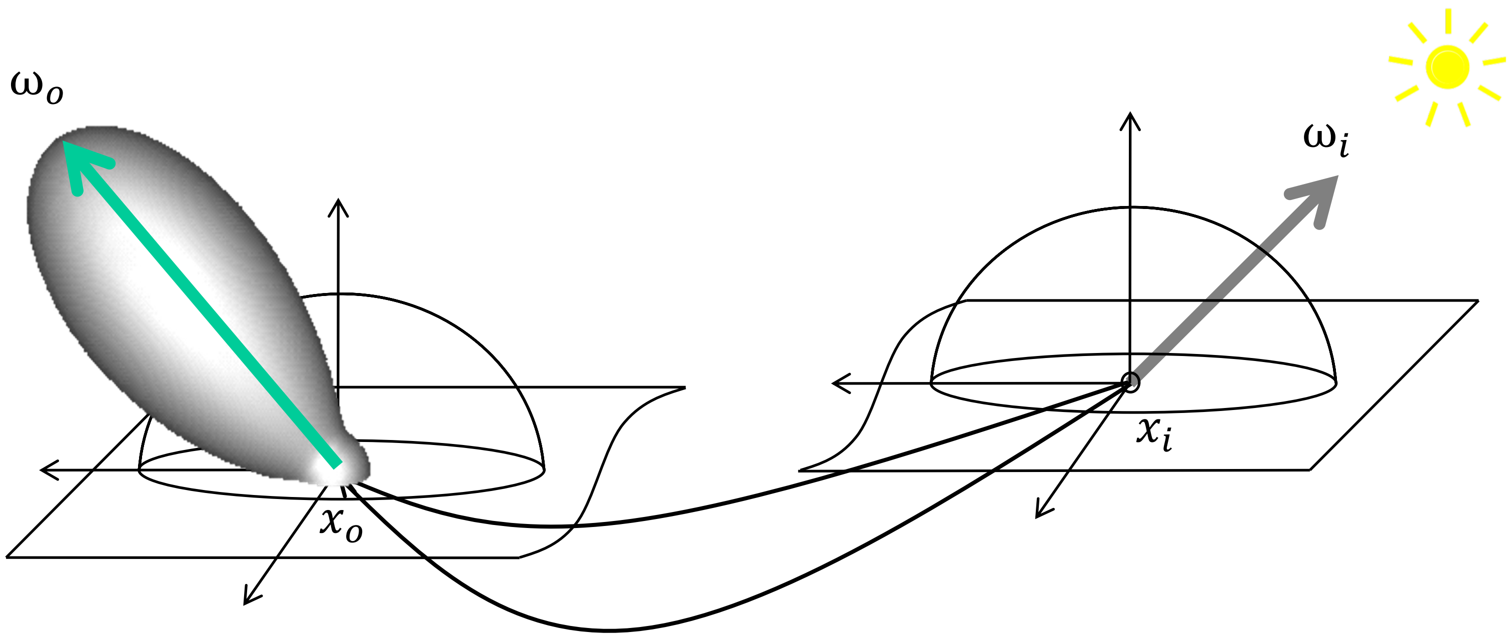
$$f_r\big((x_i, \omega_i) \rightarrow (x_o, \omega_o)\big)$$



Generalization – 9D

- **Wavelength Dependence**

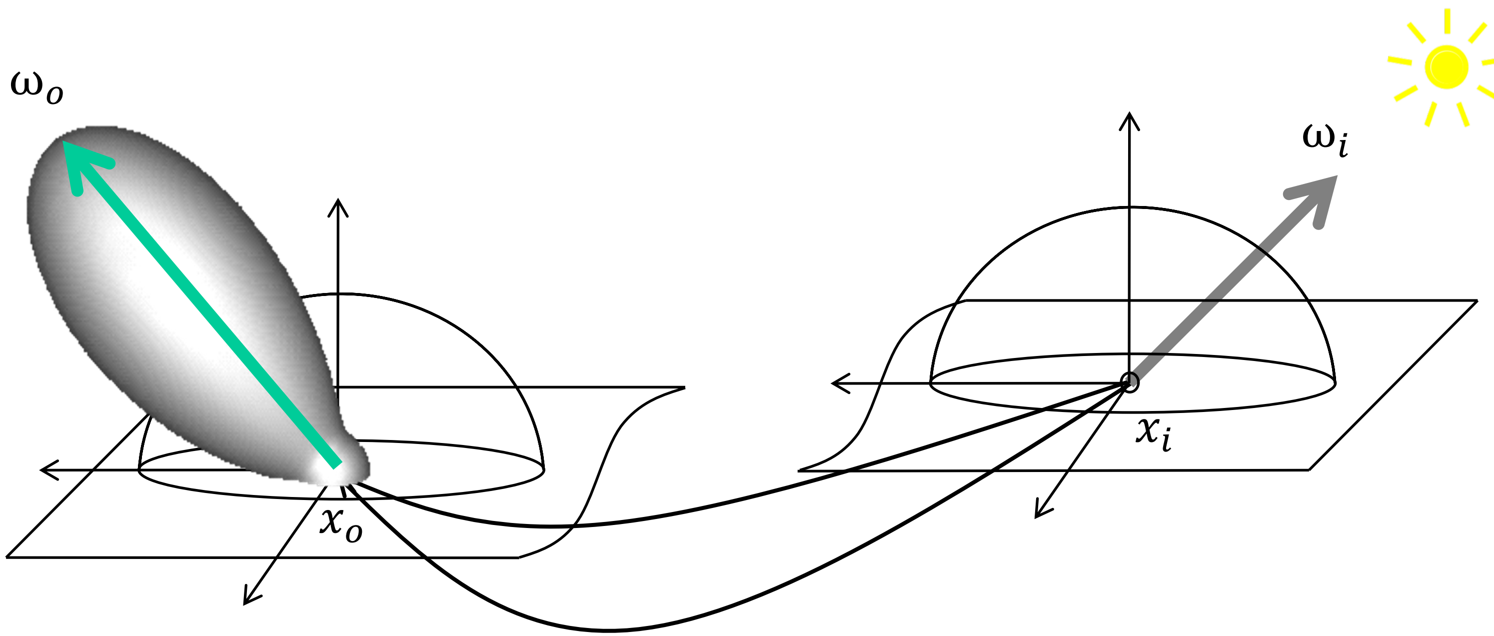
$$f_r\big(\lambda, (x_i, \omega_i) \rightarrow (x_o, \omega_o)\big)$$



Generalization – 10D

- **Fluorescence**
  - Change to longer wavelength

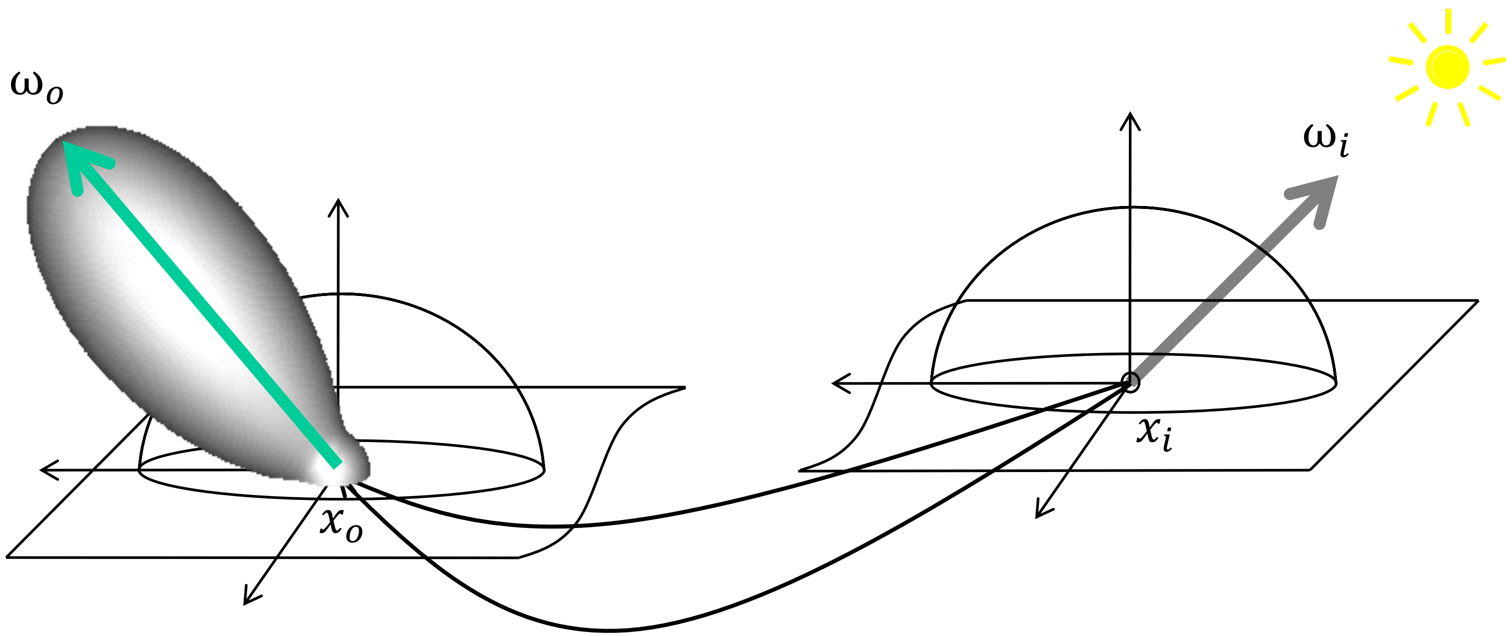
$$f_r\big((x_i, \omega_i, \lambda_i) \rightarrow (x_o, \omega_o, \lambda_o)\big)$$



Generalization – 11D

- **Time-Varying Surface Characteristics**
  - Weathering

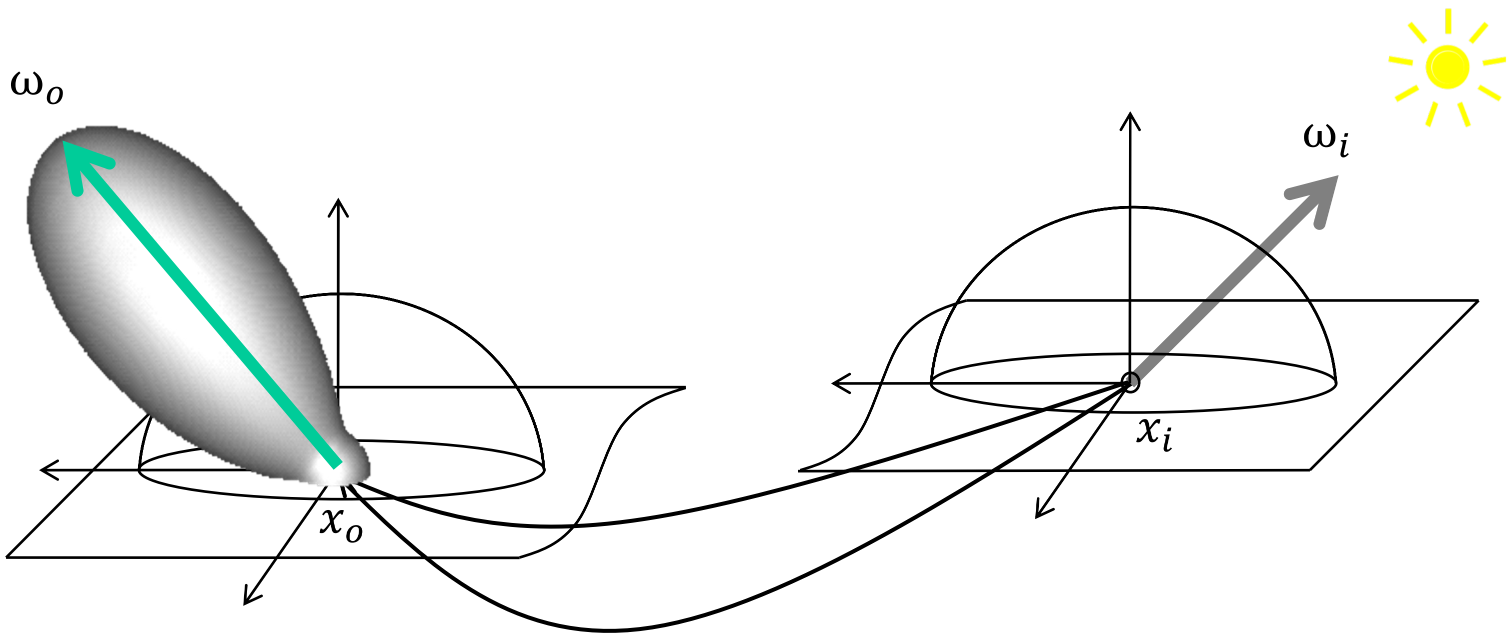
$$f_r(t, (x_i, \omega_i, \lambda_i) \rightarrow (x_o, \omega_o, \lambda_o))$$



Generalization – 12D

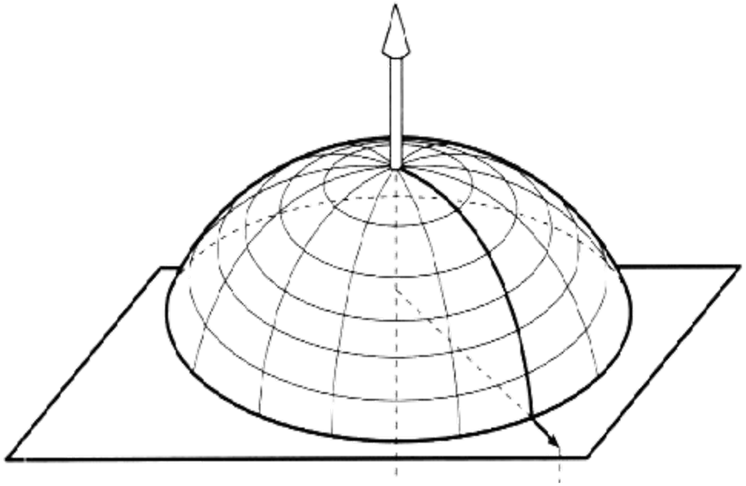
- **Phosphorescence**
  - Temporal storage of light
- **Different Path Length**

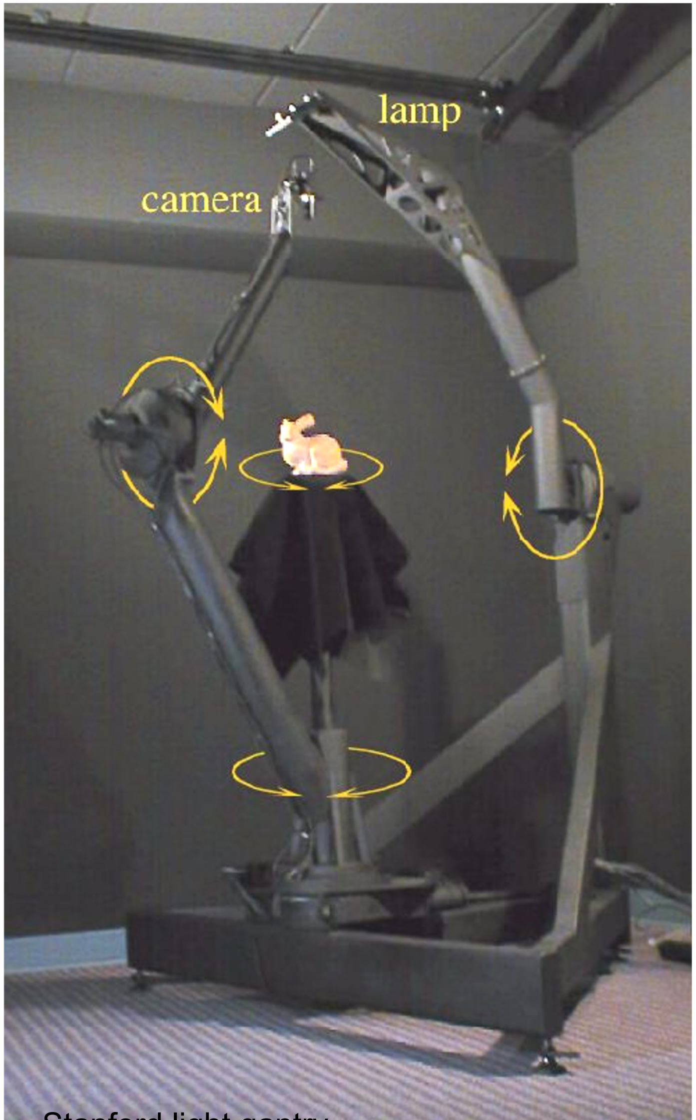
$$f_r\big((x_i, \omega_i, t_i, \lambda_i) \rightarrow (x_o, \omega_o, t_o, \lambda_o)\big)$$



BRDF Measurement

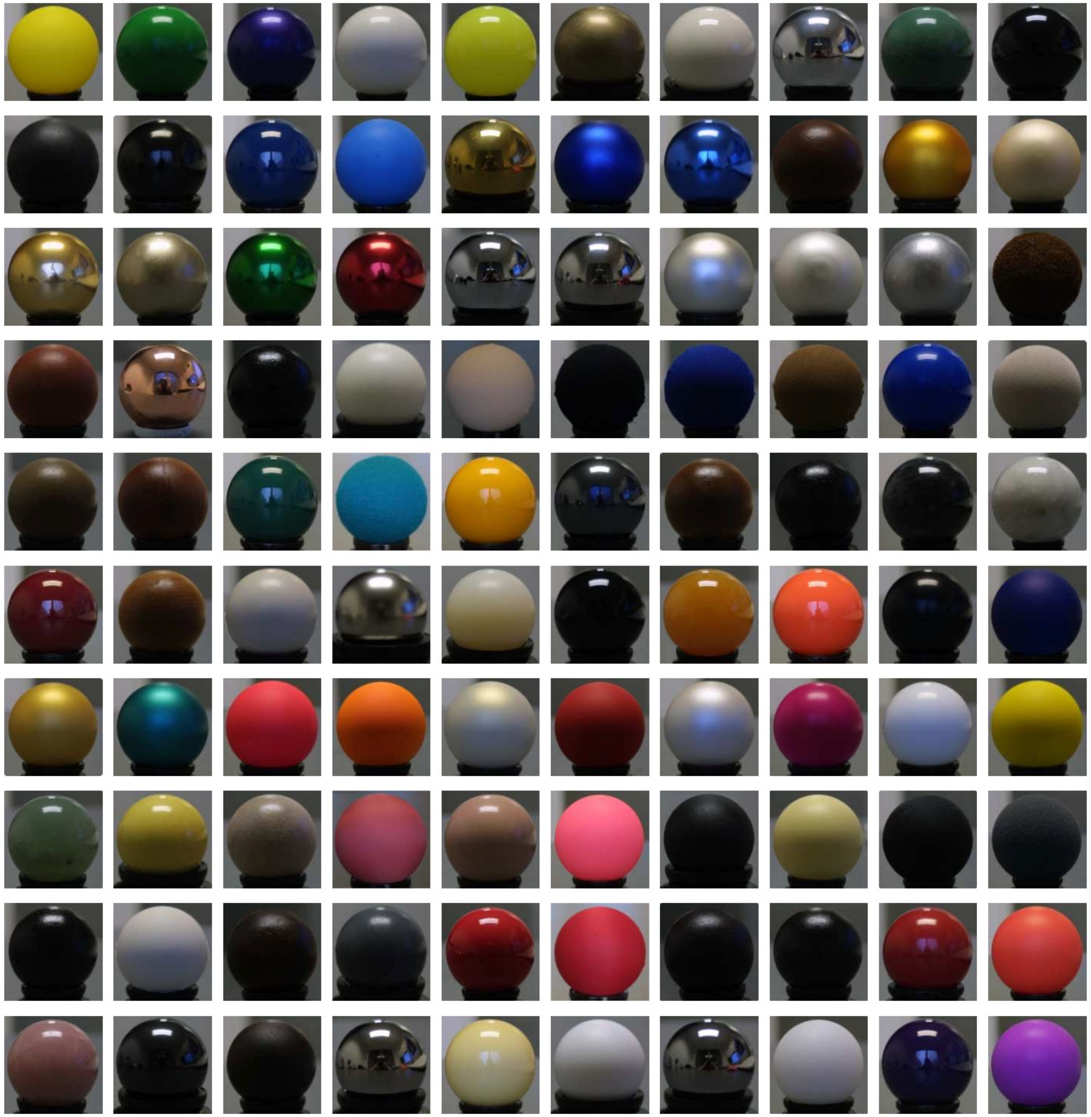
- **Gonio-Reflectometer**
  - Point light source position (θ_i, ϕ_i)
  - Light detector position (θ_o, ϕ_o)
  - 4 directional degrees of freedom
- **BRDF Representation**
  - Quadratic in incident resolution
  - Quadratic in exitant resolution
  - Quartic memory requirements!!





Stanford light gantry

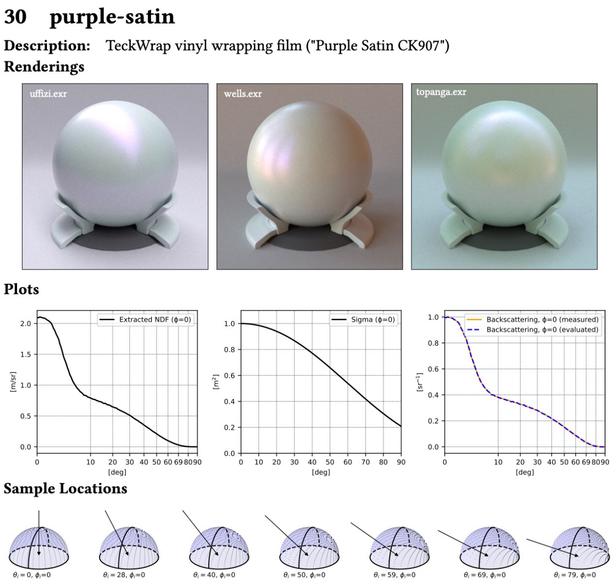
MERL Database

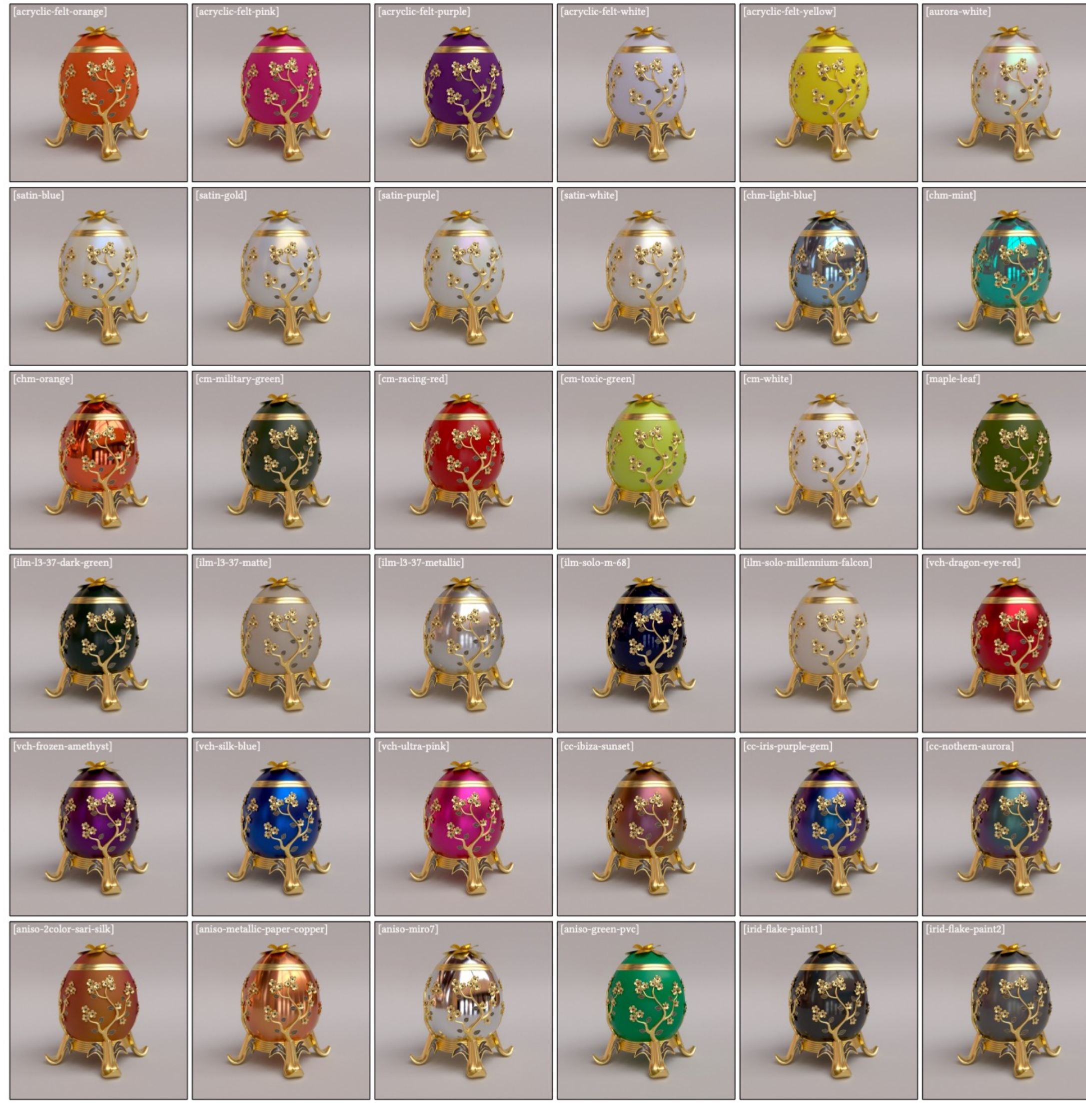


Gonio-photometer

EPFL, Switzerland

RGL Database





BRDF MODELS

Analytical Models

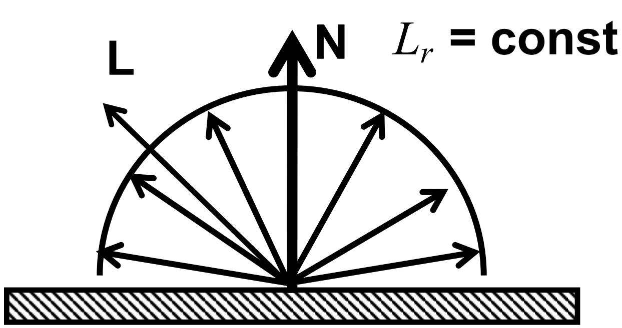
- **Storage**
  - Virtually none
- **Continuous Function**
  - No interpolation
- **Speed**
  - No data look up
  - Fast evaluation
- **Parameterization**
  - Distribution controlled by a few (intuitive) parameters

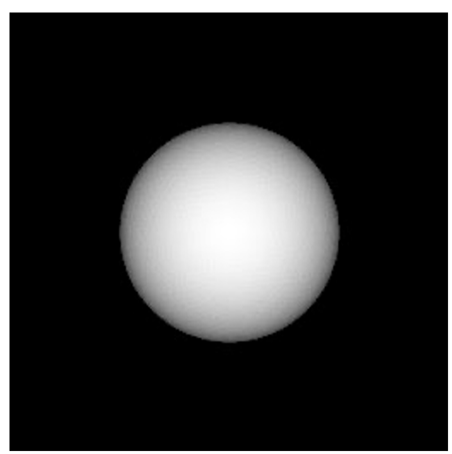
Lambertian BRDF

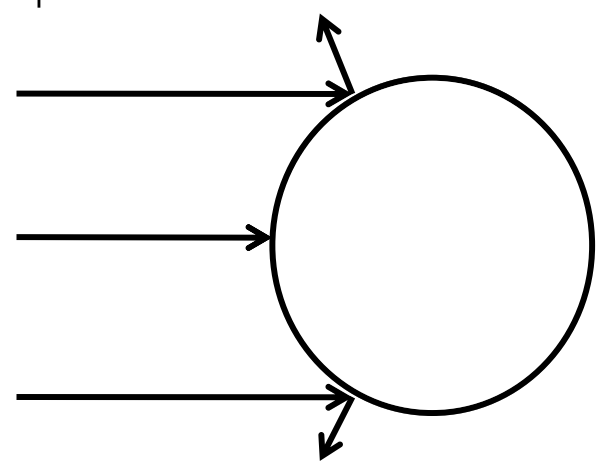
- **Ideal Diffuse Reflection**
  - Perfectly rough surfaces
- **Constant Distribution**
  - Light uniformly reflected in all output directions
- **Reflected Radiance**
  - Independent of viewing direction
  - Depends only on illumination direction

$$f_r(x, \omega_i, \omega_o) = \frac{1}{\pi}$$

$$L_r(x, \omega_o) = \frac{1}{\pi} \int_{\Omega_+} L_i(x, \omega_i) \cos\theta_i \, d\omega_i = \frac{E}{\pi}$$







Lambertian BRDF

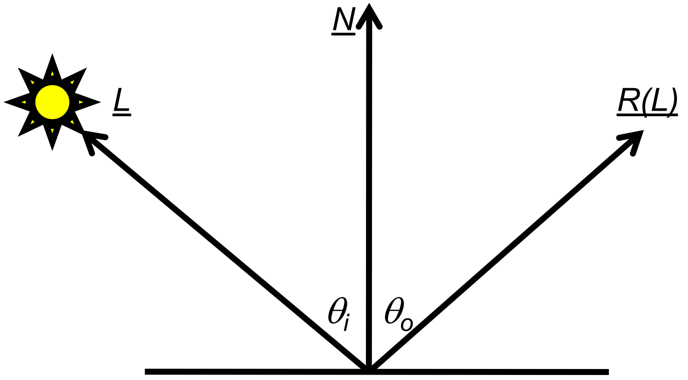
- **Theoretical Model**
  - Assuming all photons undergo multiple scattering events
- **Experimental Approximations**
  - Pressed magnesium oxide powder
  - Almost never valid at high angles of incidence
  - Hard to manufacture ideal diffuse paints

Mirror BRDF

- **Ideal Specular Reflection**
  - Perfectly smooth surface
- **Dirac Delta Distribution**
  - Light entirely reflected in a single output direction

$$f_r(x, \omega_i, \omega_o) = F_r(\theta_i) \frac{\delta(\omega_i - R(\omega_o))}{\cos\theta_i}$$

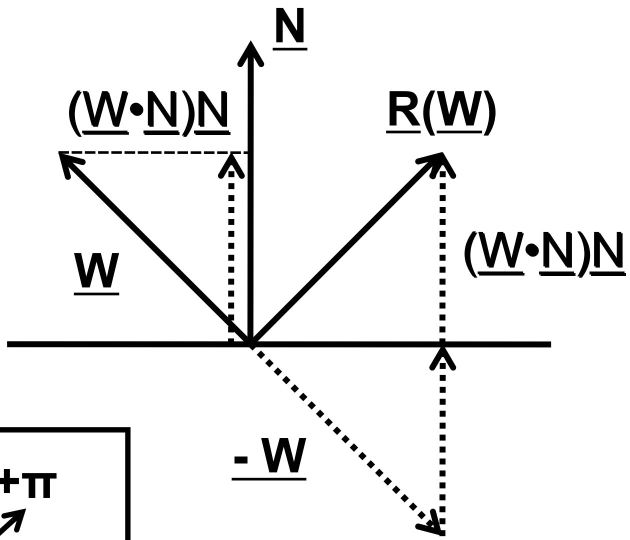
- **Reflected Radiance**
  - Independent of illumination direction
    - No need to loop over light sources
  - Depends only on viewing direction
    - Trace 1 secondary ray instead
    - Up to some level of recursion!

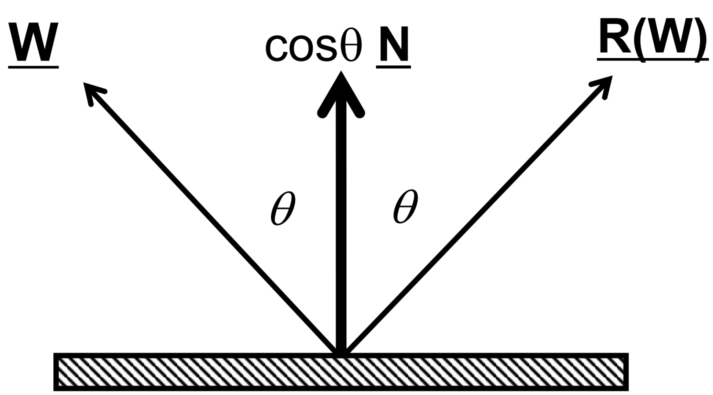


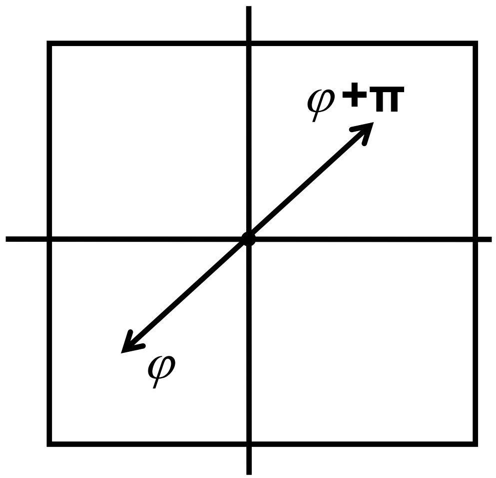
$$L_r(x, \omega_o) = \int_{\Omega_+} L_i(x, \omega_i) F_r(\theta_i) \frac{\delta(\omega_i - R(\omega_o))}{\cos\theta_i} \cos\theta_i \, d\omega_i = L_i(x, R(\omega_o)) F_r(\theta_o)$$

Reflection Direction

- **Properties**
  - Contained in a plane with incident ray and surface normal vector
  - Angle of reflectance equal to angle of incidence
- **Computation**
  - $\underline{R}(\underline{W}) + \underline{W} = 2\cos\theta\,\underline{N} = 2(\underline{W}\bullet\underline{N})\,\underline{N}$
  - $\underline{R}(\underline{W}) = 2(\underline{W}\bullet\underline{N})\,\underline{N} - \underline{W}$

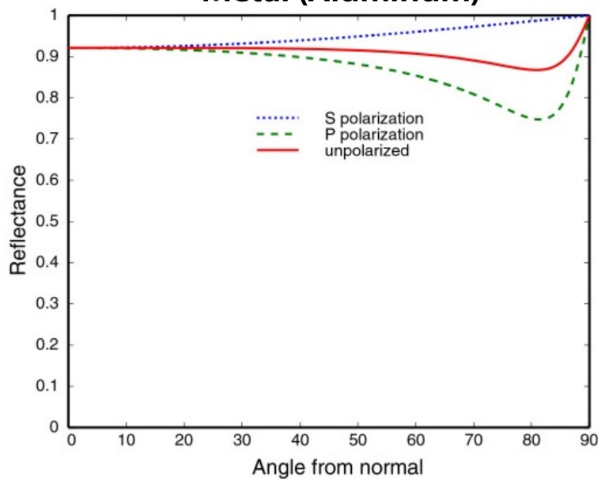




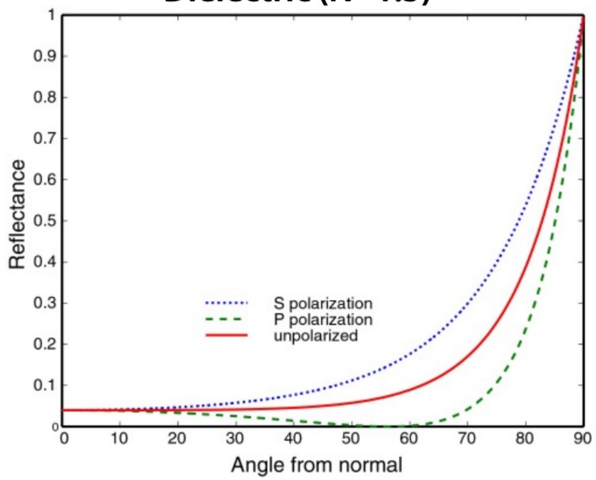


Fresnel

Metal (Aluminum)



Dielectric (N=1.5)



Gold	**F(0)=0.82**
Silver	**F(0)=0.95**

Glass	**n=1.5 F(0)=0.04**
Diamond	**n=2.4 F(0)=0.15**

Fresnel Term for Conductors

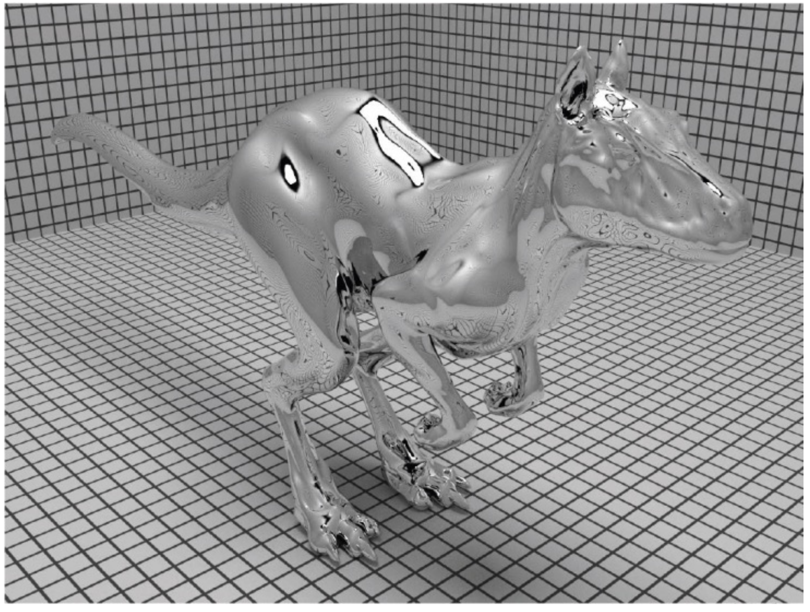
- **Electric Conductors (e.g. Metals)**
  - Reflect light according to Fresnel formula
  - Non-reflected light is absorbed
- **Parameters**
  - Index of refraction η
  - Absorption coefficient κ

Object	η	k
Gold	0.370	2.820
Silver	0.177	3.638
Copper	0.617	2.63
Steel	2.485	3.433

- **Parallel & Perpendicular Polarized Light**

$$r_{\parallel}^2 = \frac{(\eta^2 + k^2)\cos^2\theta_i - 2\eta\cos\theta_i + 1}{(\eta^2 + k^2)\cos^2\theta_i + 2\eta\cos\theta_i + 1}$$

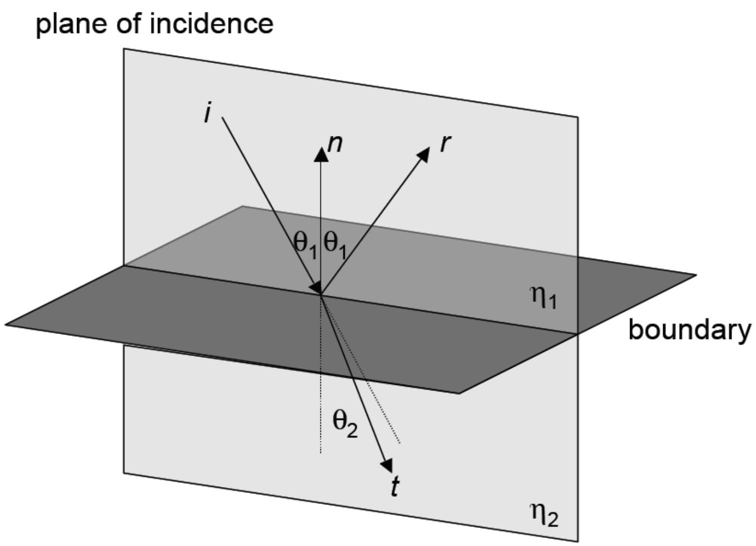
$$r_{\perp}^2 = \frac{(\eta^2 + k^2) - 2\eta\cos\theta_i + \cos^2\theta_i}{(\eta^2 + k^2) + 2\eta\cos\theta_i + \cos^2\theta_i}$$



- **Non-Polarized** $F_r = \frac{1}{2}(r_{\parallel}^2 + r_{\perp}^2)$
  - Fr in [0, 1]

Glass BSDF

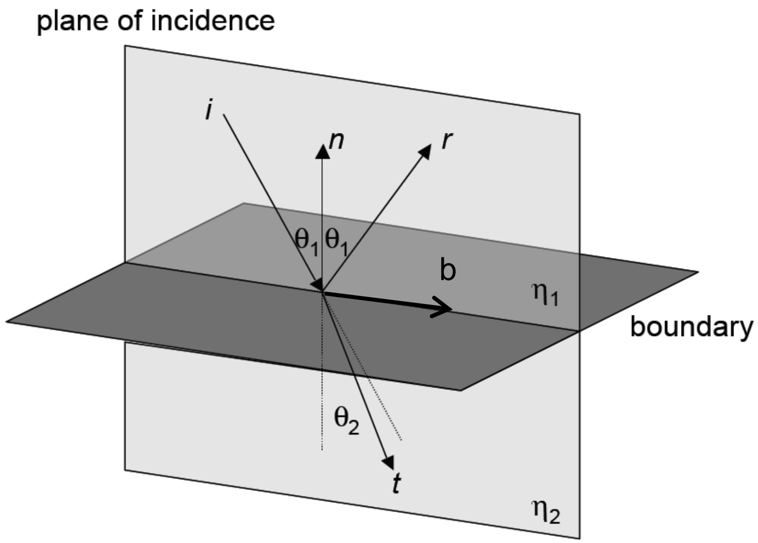
- **Ideal Specular Reflection/Refraction**
  - Perfectly smooth surface
- **Dirac Delta Distribution**
  - Light entirely redirected in two output directions
- **Index of Refraction**
  - Speed of light in vacuum c
  - Speed of light in material v
  - n = c / v
- **Reflected Radiance**
  - Independent of illumination direction
    - No need to loop over light sources
  - Depends only on viewing direction
    - Trace 2 secondary rays instead
    - Up to some level of recursion!

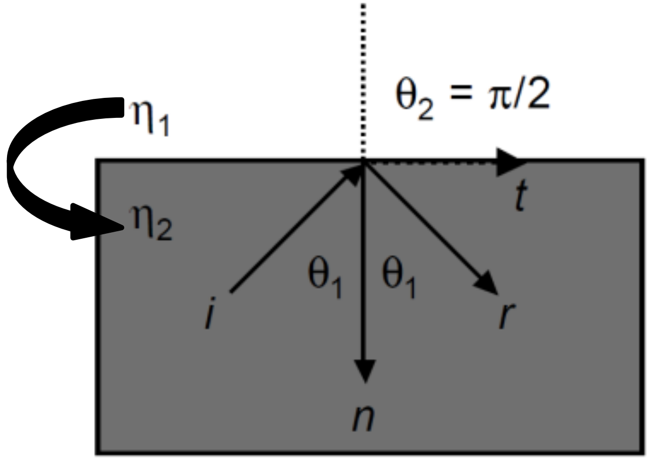


$$L_r(x,\omega_o) = L_i\big(x, R(\omega_o)\big)F_r(\theta_o) + L_i(x, T(\omega_o))F_t(\theta_o)\frac{n_o^2}{n_i^2}$$

Refraction Direction

- **Snell's Law**
  - $n_1 \sin\theta_1 = n_2 \sin\theta_2$
  - $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$
  - $\vec{t} = \sin\theta_2\ \vec{b}\ - \cos\theta_2\ \vec{n}$
- **Total Internal Reflection**
  - $\sin\theta_1 > \frac{n_2}{n_1}$ would yield $\sin\theta_2 > 1$
  - Then no refraction occurs
  - All light is internally reflected





Fresnel Term for Dielectrics

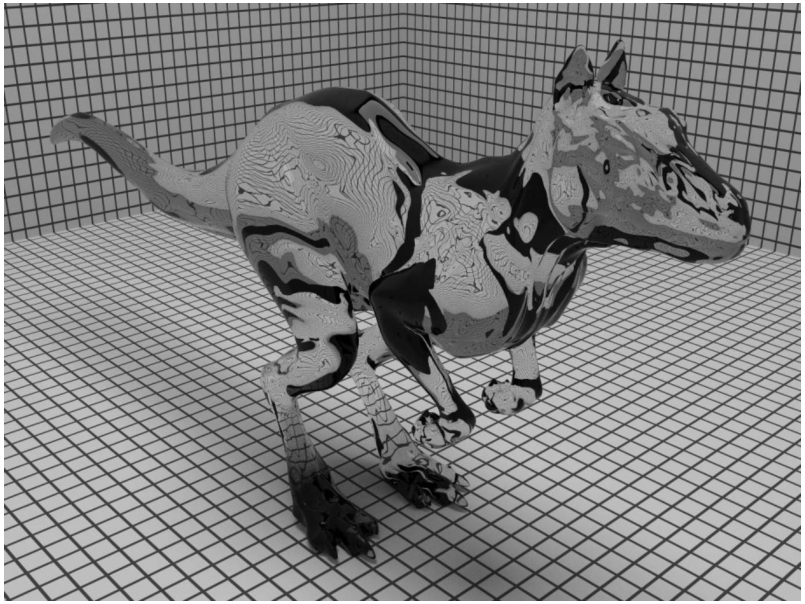
- **Dielectrics (e.g. Glass)**
  - Reflect light according to Fresnel formula
  - Rest is transmitted: $F_t = 1 - F_r$

Medium	Index of refraction η
Vacuum	1.0
Air at sea level	1.00029
Ice	1.31
Water (20° C)	1.333
Fused quartz	1.46
Glass	1.5–1.6
Sapphire	1.77
Diamond	2.42

- **Parameters**
  - Ref. index in incident medium η_i
  - Ref. index in transmitted medium η_t
- **Parallel & Perpendicular Polarized Light**

$$r_{\parallel} = \frac{\eta_t \cos\theta_i - \eta_i \cos\theta_t}{\eta_t \cos\theta_i + \eta_i \cos\theta_t}$$

$$r_{\perp} = \frac{\eta_i \cos\theta_i - \eta_t \cos\theta_t}{\eta_i \cos\theta_i + \eta_t \cos\theta_t},$$

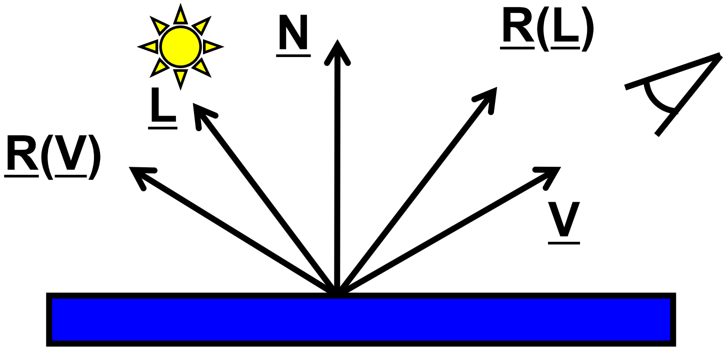


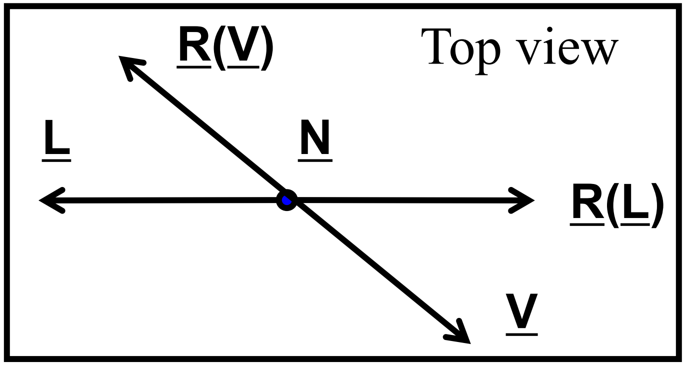
- **Non-Polarized** $F_r = \frac{1}{2}(r_{\parallel}^2 + r_{\perp}^2)$
  - Fr in [0, 1]

Phong BRDF

- **Glossy Reflection**
  - Shiny surface due to variable roughness
- **Phenomenological Distribution**
  - Light mainly reflected around reflection direction
- **Reflected Radiance**
  - Depends on viewing direction
  - Depends on illumination direction

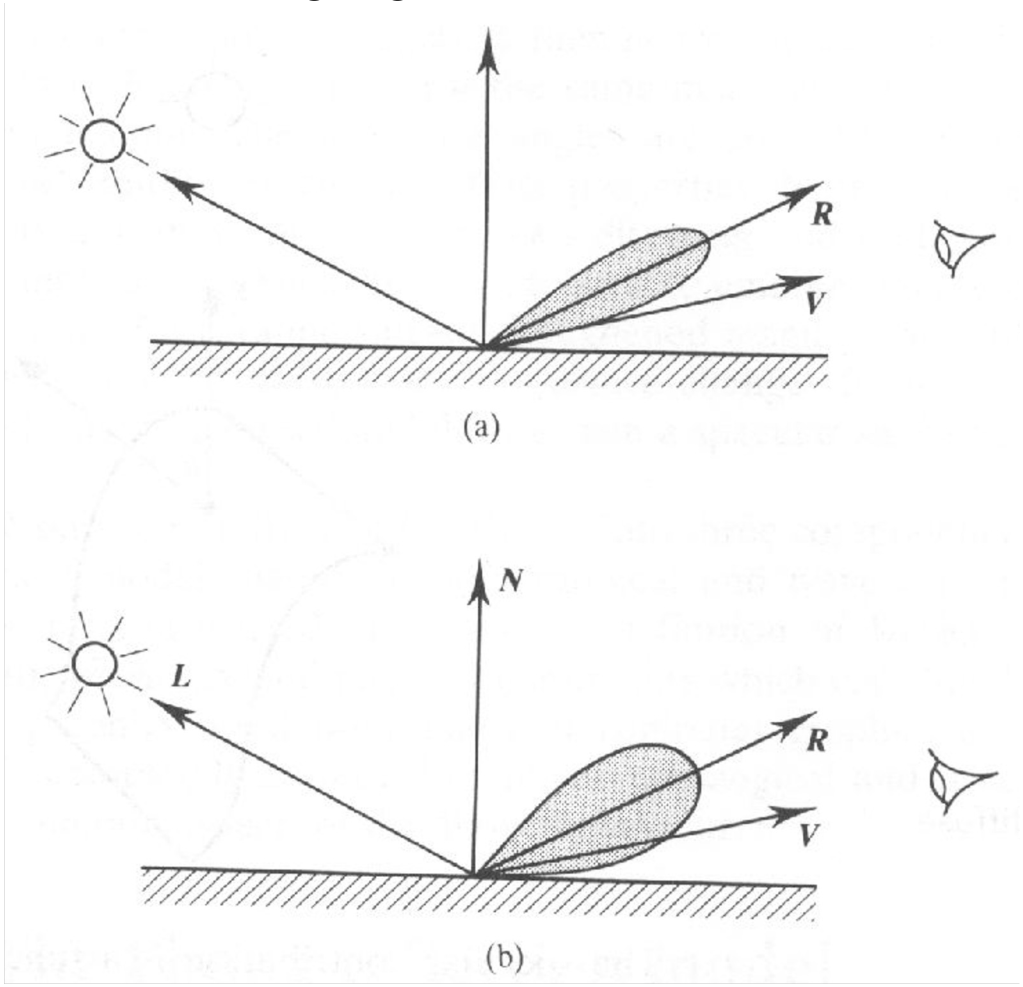
$$f_r(x, \omega_i, \omega_o) = \frac{\alpha + 2}{2\pi} \max\big(0, w_i \cdot R(w_o)\big)^{\alpha}$$





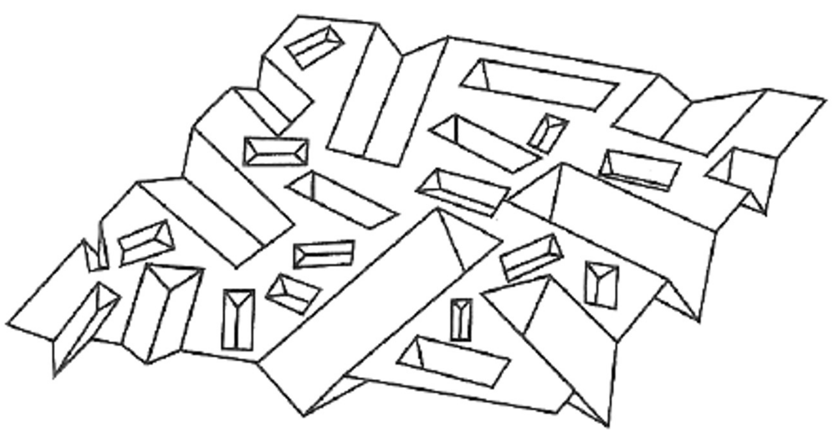
Phong Exponent

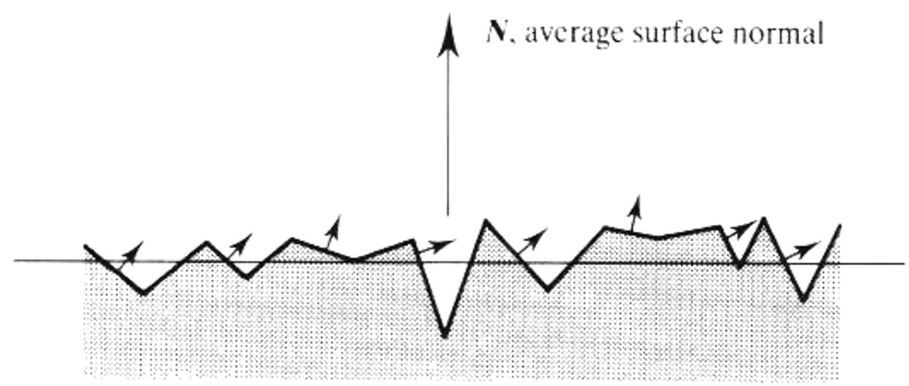
- **Cosine Lobe Spread**
  - Determines size of highlight



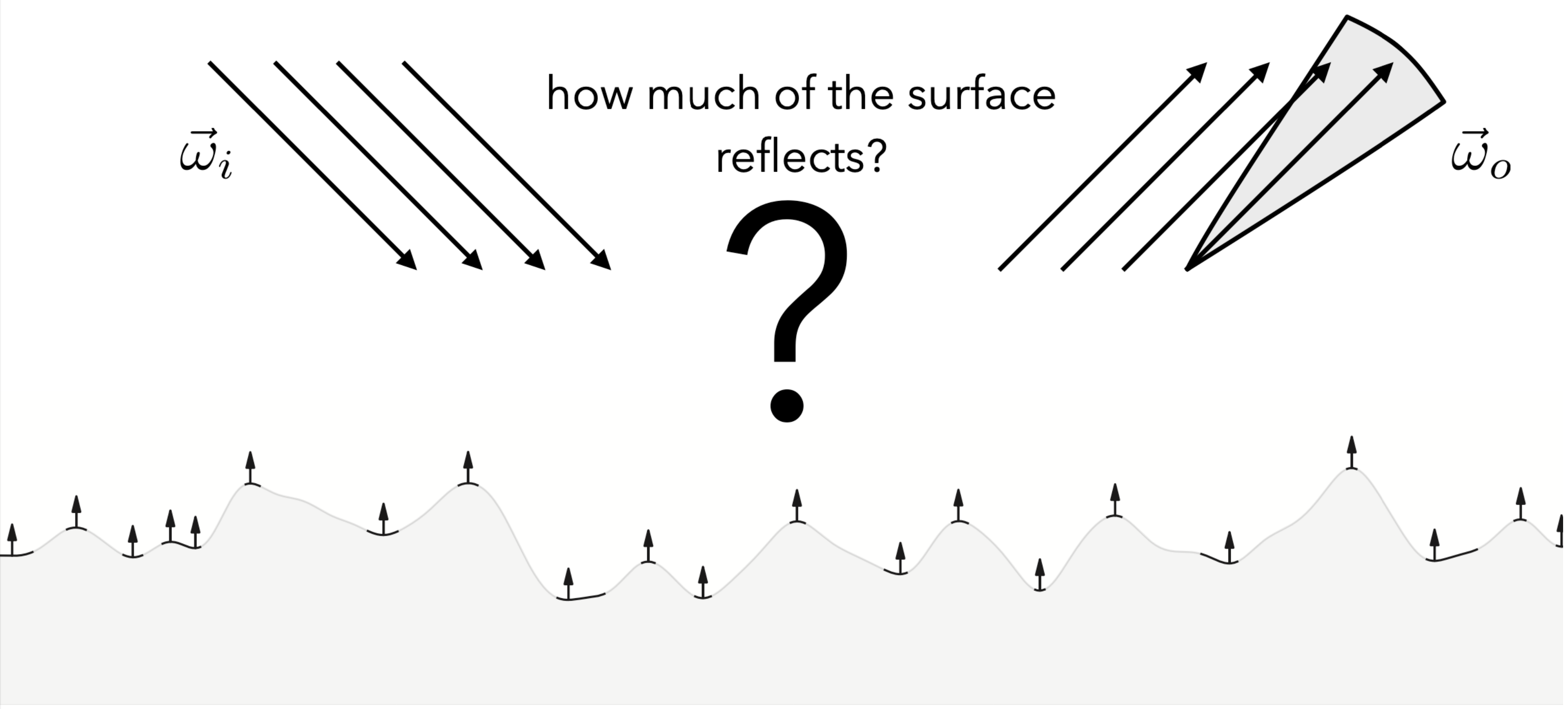
Cook-Torrance BRDF

- **Glossy Reflection**
  - Shiny surface due to variable roughness
- **Physical Distribution**
  - Assume surface is composed of perfectly specular microfacets
  - Light mainly reflected around off-specular reflection direction
- **Reflected Radiance**
  - Depends on viewing direction
  - Depends on illumination direction



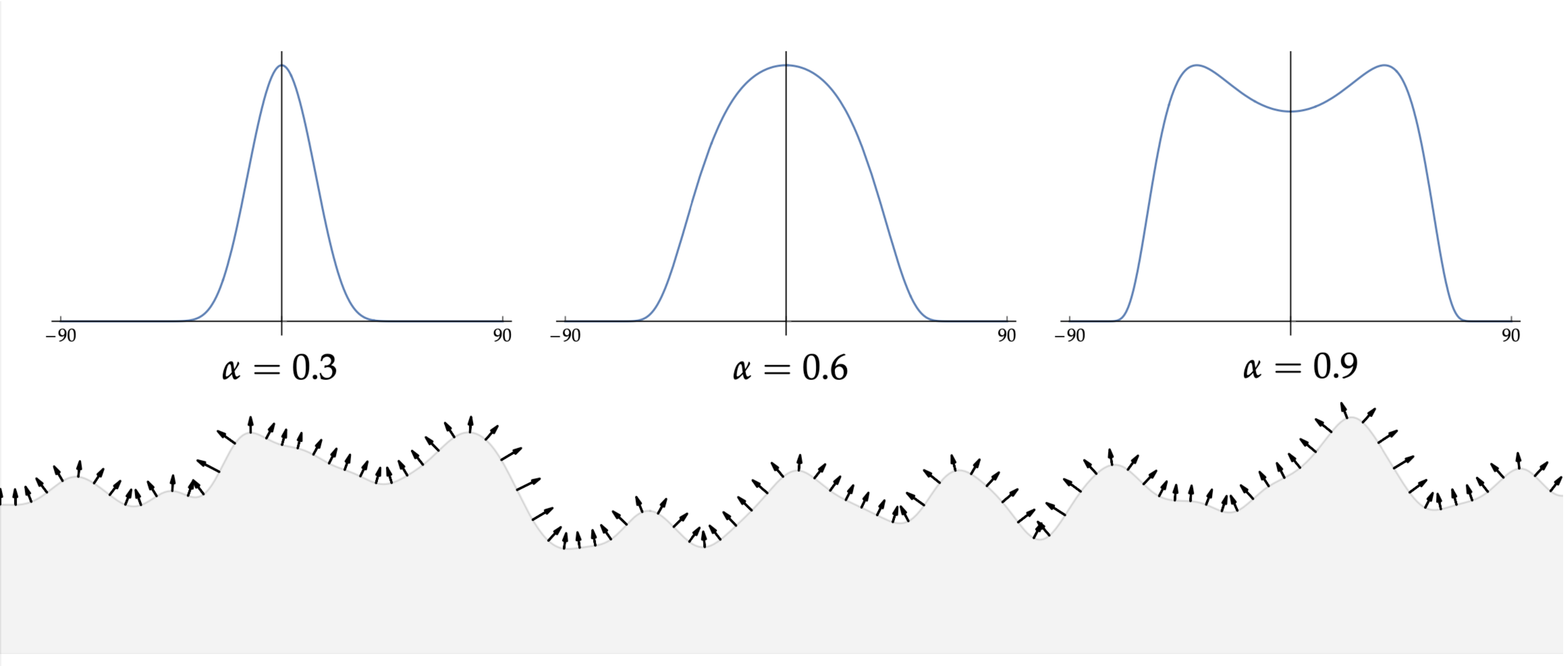


Microfacet Distribution



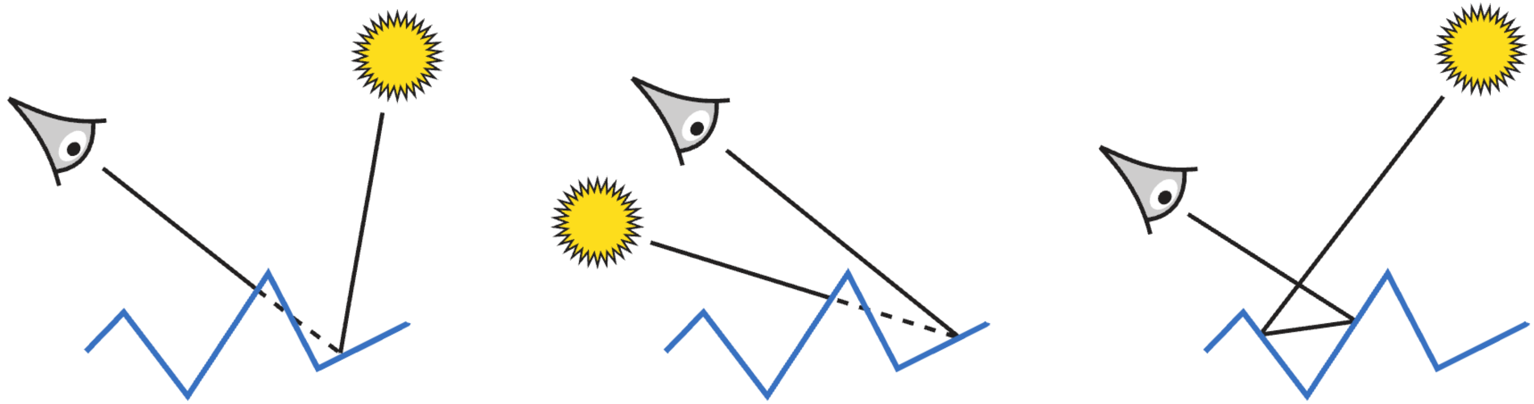
Microfacet Distribution

- **Models the statistical distribution of slopes**
- **Roughness controls the shape of the distribution**



Shadowing and Masking

- **Microfacets can be *shadowed* and/or *masked***



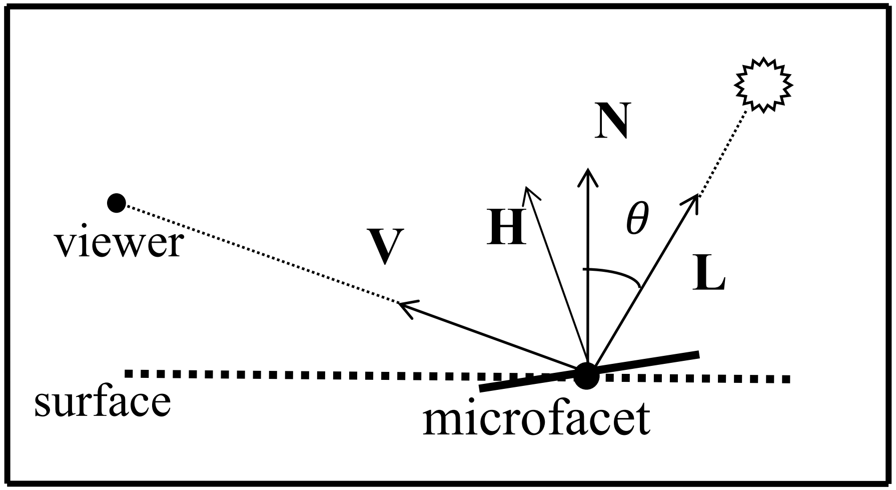
(a) **Masking** (b) **Shadowing** (c) **Interreflection**

Cook-Torrance BRDF

- **Formulation**

$$f_r(x, \omega_i, \omega_o) = \frac{D(\omega_h)\mathrm{G}(\omega_h, \omega_i, \omega_o)F_r(\angle\omega_o, \omega_h)}{4(n \cdot \omega_o)(n \cdot \omega_i)}$$

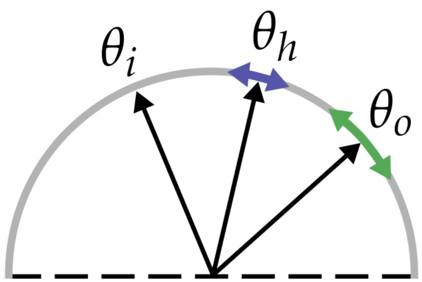
  - D : Microfacet distribution
  - G : Geometric attenuation factor
  - F_r : Fresnel term
  - $n \cdot \omega_o$: Accounts for grazing viewing angles
  - $n \cdot \omega_i$: Accounts for grazing illumination angles



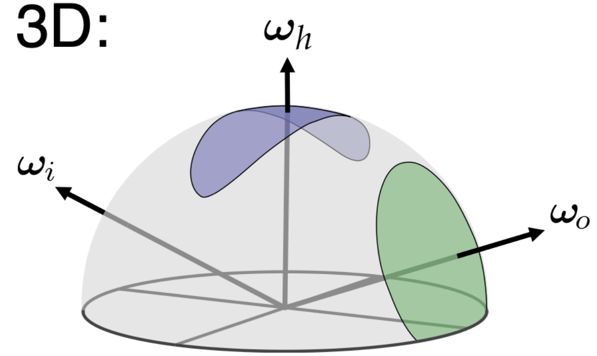
Half-direction transformation

- **Mirror Microfacets**
  - Light from ω_i reflected into ω_o
  - Only by facets whose normal ω_h is halfway between ω_i and ω_o
  - Also called the half-vector or halfway vector

2D:



3D:



$$\theta_h := \frac{\theta_i + \theta_o}{2}$$

$$\frac{\mathrm{d}\theta_h}{\mathrm{d}\theta_o} = ?$$

$$\boldsymbol{\omega}_h := \frac{\boldsymbol{\omega}_i + \boldsymbol{\omega}_o}{\|\boldsymbol{\omega}_i + \boldsymbol{\omega}_o\|}$$

$$\frac{\mathrm{d}\boldsymbol{\omega}_h}{\mathrm{d}\boldsymbol{\omega}_o} = \frac{1}{4|\boldsymbol{\omega}_o \cdot \boldsymbol{\omega}_h|}$$

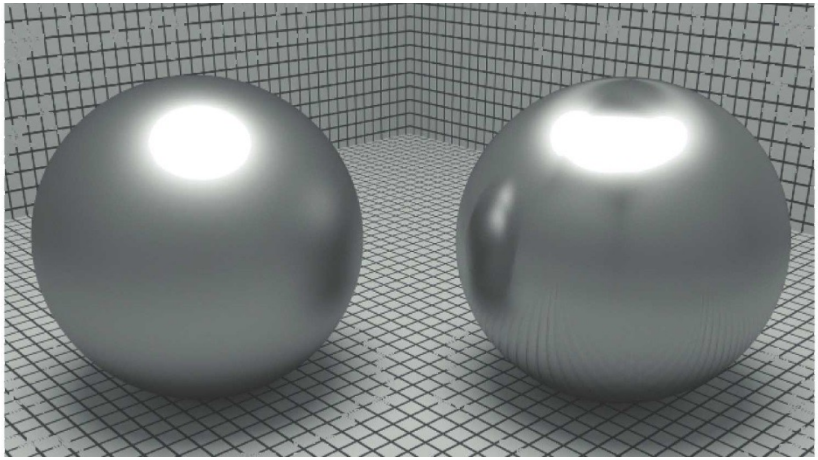
Microfacet Distribution

- **Definition**
  - Statistical distribution of microfacet orientations
- **Blinn Microfacet Distribution**
  - Isotropic collection of microfacets
    - Only depends on $\angle w_h, n$
  - Cosine power lobe
    - Exponent controls roughness/specularity

$$D(\omega_h) = \frac{e+2}{2\pi}\,(w_h \cdot n)^e$$

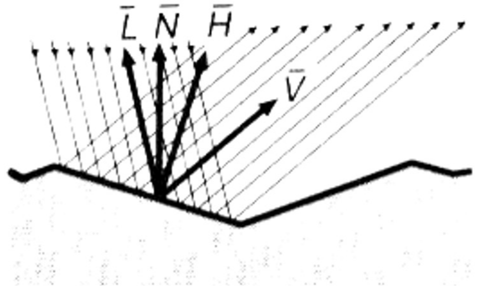
- **Extension**
  - Anisotropic collection of microfacets
    - ϕ_h determined by “tangent” vector

$$D(\omega_\mathrm{h}) = \frac{\sqrt{(e_x+2)(e_y+2)}}{2\pi}(\omega_\mathrm{h} \cdot \mathbf{n})^{e_x \cos^2\phi + e_y \sin^2\phi}$$



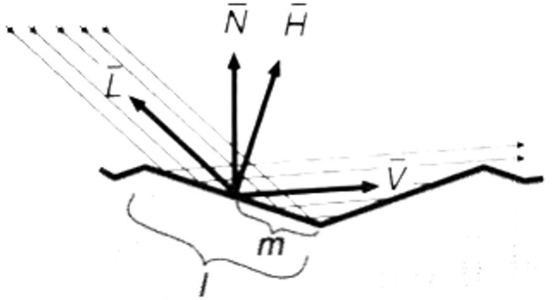
Geometric Attenuation Factor

- **Definition**
  - Models self-masking and shadowing effects of microfacets
  - Assumes V-shaped grooves
- **Fully illuminated and visible**



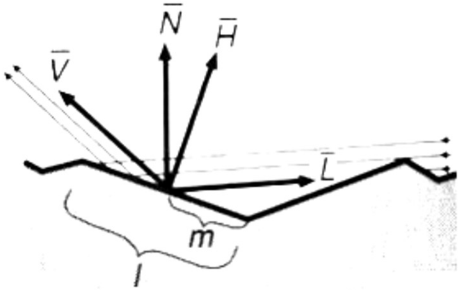
$$G = 1$$

- **Partial masking of reflected light**



$$G = \frac{2(n \cdot \omega_h)(n \cdot \omega_o)}{(\omega_o \cdot \omega_h)}$$

- **Partial shadowing of incident light**



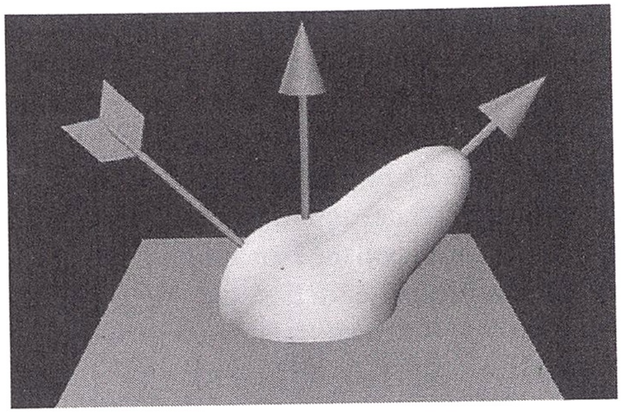
$$G = \frac{2(n \cdot \omega_h)(n \cdot \omega_i)}{(\omega_i \cdot \omega_h)}$$

- **Final**

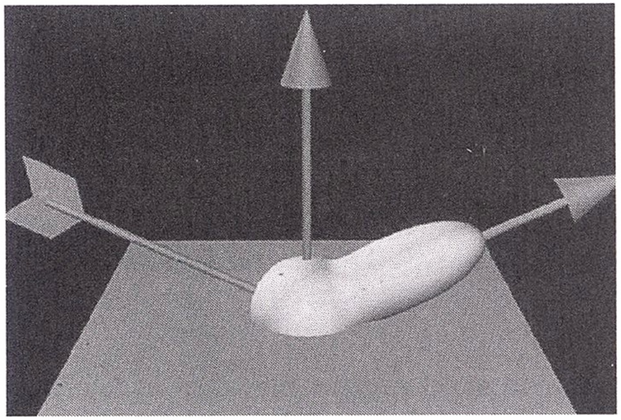
$$G = \min\left\{1, \frac{2(n \cdot \omega_h)(n \cdot \omega_o)}{(\omega_o \cdot \omega_h)}, \frac{2(n \cdot \omega_h)(n \cdot \omega_i)}{(\omega_i \cdot \omega_h)}\right\}$$

Phong vs. Cook-Torrance

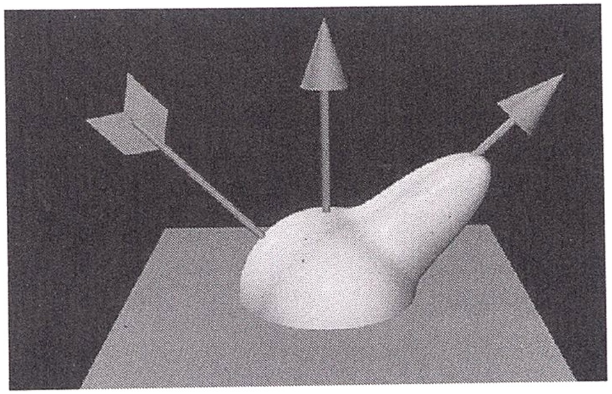
- **Off-Specular Lobe**



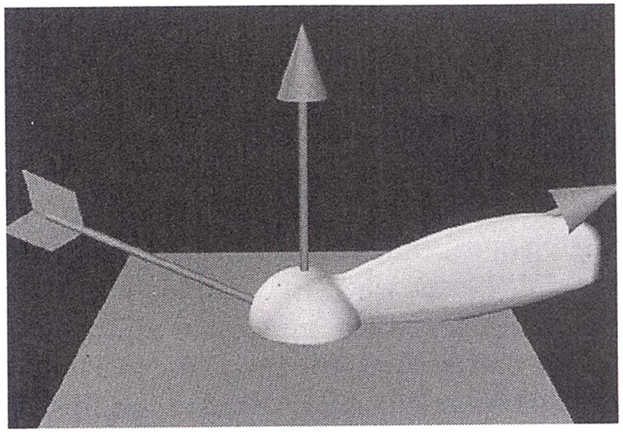
(a)



(b)



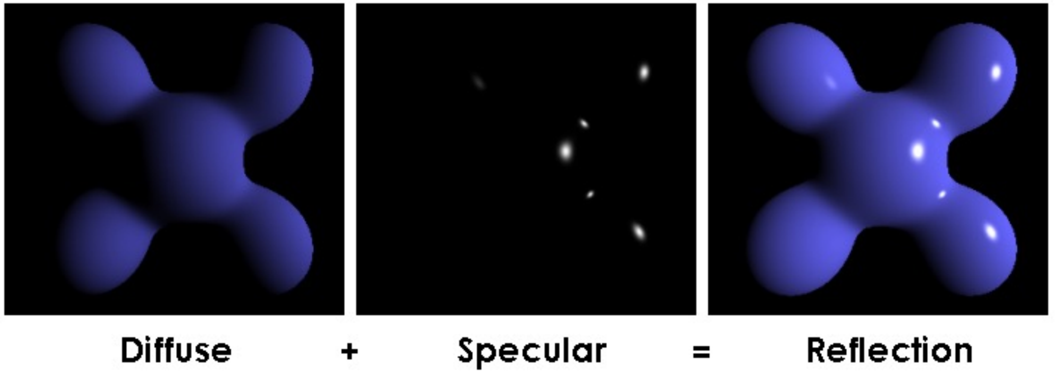
(c)



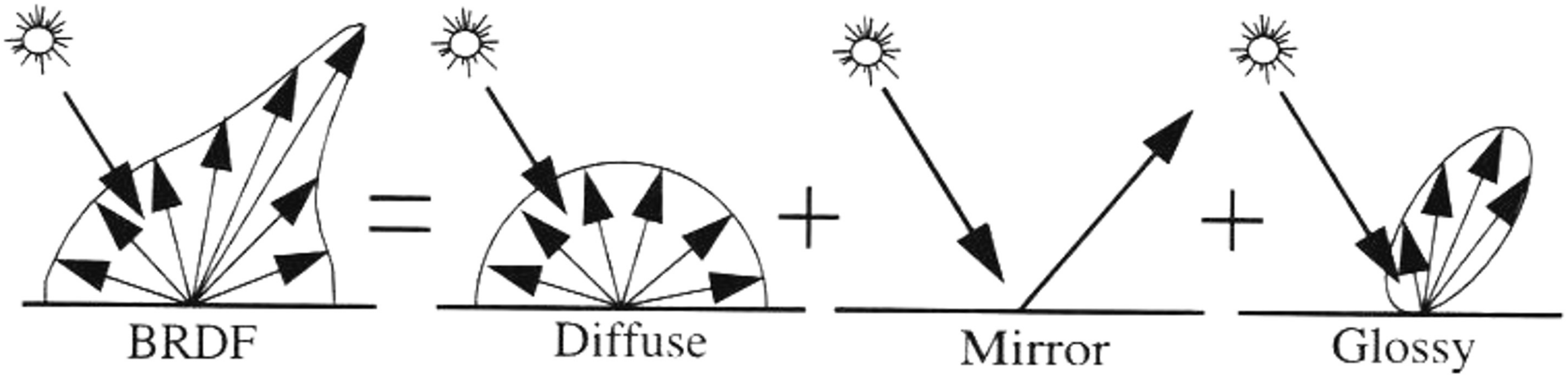
(d)

Combining BRDFs

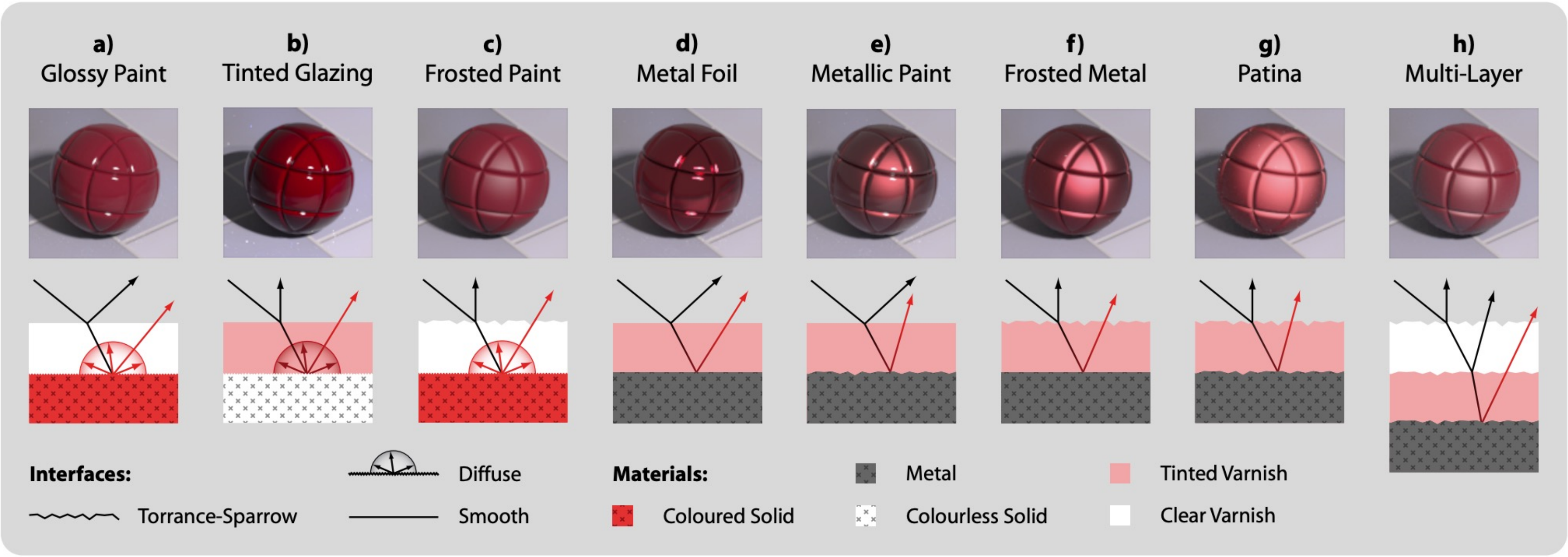
- **Linear Combination**
  - Combine simple BRDFs
  - Model complex BRDF
  - Wavelength-dep. weights
- **Application**
  - Diffuse, specular and glossy terms
  - Energy conservation: $k_d(x) + k_s(x) + k_g(x) \leq 1$



$$f_r(x, \omega_i, \omega_o) = k_d(x) f_{rd}(x, \omega_i, \omega_o) + k_s(x) f_{rs}(x, \omega_i, \omega_o) + k_g(x) f_{rg}(x, \omega_i, \omega_o)$$



Multi-Layered Materials



Weidlich et al. [2007], Arbitrarily Layered Micro-Facet Surfaces

Pearlescent Materials

Guillén et al. [2020], A General Framework for Pearlescent Materials