# Computer Graphics

## HDR Imaging

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# Overview

- **HDR Acquisition**
- **Tone-Mapping**

# High Dynamic Range Imaging

- **Contrast Handling**
  - Input: HDR intensities in real-world scenes (e.g., from rendering)
  - Output: Typically, LDR devices
- **Acquisition of HDR input**
  - LDR cameras
    - Requires multiple exposures to fully cover the high dynamic range
    - Now often integrated (e.g., into smartphones)
  - HDR cameras
    - Still rather exotic
    - AI in smartphones adds HDR capabilities (e.g., night mode)
- **Display**
  - Display on LDR monitors
    - *Tone mapping* to perceptively compress HDR to LDR
  - HDR displays
    - Modern displays are now getting more and more HDR capable
    - Sometime >1000 individual LED in backlight or micro-LED display

Part I

# HDR Acquisition

# Acquisition of HDR from LDR

- **Limited dynamic range of cameras is a problem**
  - Shadows are underexposed
  - Bright areas are overexposed
  - Sensor's temporal sampling density is not sufficient → saturation
- **Good sign**
  - Some modern CMOS imagers have a higher (and often sufficient) dynamic range than most traditional CCD sensors

- **Basic idea of multi-exposure techniques**
  - Combine multiple images with different exposure settings
  - Makes use of available sequential dynamic range
- **Other techniques available**
  - E.g., HDR video

# Exposure Bracketing

- **Acquiring HDR from LDR input devices**
  - Take multiple photographs with different times of exposure

- **Issues**
  - How many exposure levels?
  - How much difference between exposures?
  - How to combine them?

# Application

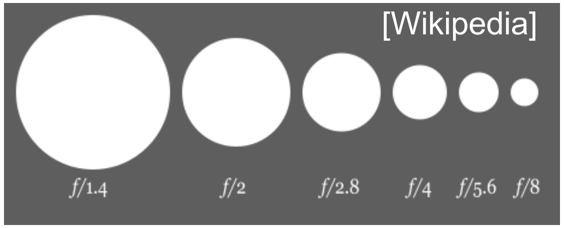
- **Capture HDR env. maps from series of input images**

1/2,000s 1/500s 1/125s 1/30s 1/8s

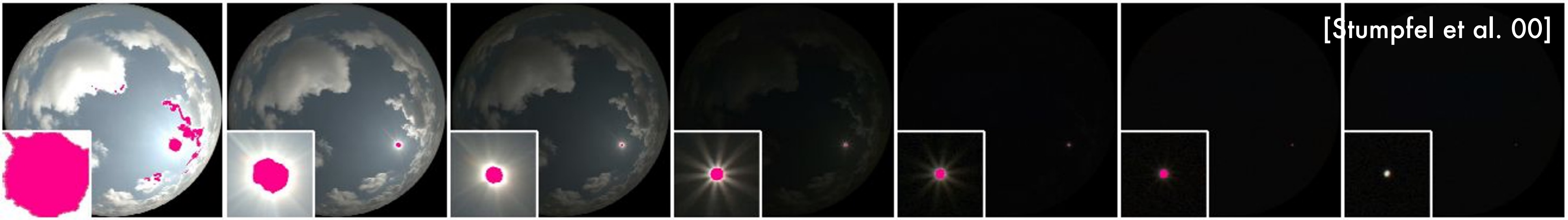
- **Used to illuminate virtual scenes with real-world environment**

# HDR in Real World Images

- **In photography**
  - F-number = focal length / aperture diameter
  - 1 f-stop incr.: f-# * √2 → aperture area / 2
- **Natural scenes**
  - 37 stops (~10 orders of magnitude)
  - 18 stops ($2^{18}$ = ~262 000) at given time of day
- **Humans**
  - After adaptation: 30 stops (~9 orders of magnitude)
  - Simultaneously: 17 stops (~4 orders of magnitude)
- **Analog cameras**
  - 10-16 stops (~3 orders of magnitude)
  - Fish-eye pix of sky with diff. exposures show saturation (e.g., sun)




Doubling the f-number decreases the aperture area by a factor of four (i.e., need to quadruple exposure time to preserve same brightness)




f/4, 1s | f/4, $\frac{1}{4}$s | f/4, $\frac{1}{32}$s | f/16, $\frac{1}{16}$s | f/16, $\frac{1}{125}$s | f/16, $\frac{1}{1000}$s | f/16, $\frac{1}{8000}$s

# Dynamic Range of Cameras

- **E.g. photographic camera with standard CCD sensor**
  - Dynamic range of sensor 1:1,000
  - Exposure time (handheld cam.): 1/60s – 1/6,000s 1:100
  - Varying aperture: f/2.0 – f/22.0 1:100 (appro.)
  - Electronic: exposure bias / varying "sensitivity" 1:10
  - Total (sequential) dynamic range 1:100,000,000

- **But simultaneous dynamic range still only 1:1,000**
  - Aperture: varying depth of field
  - Exposure time: only works for static scenes

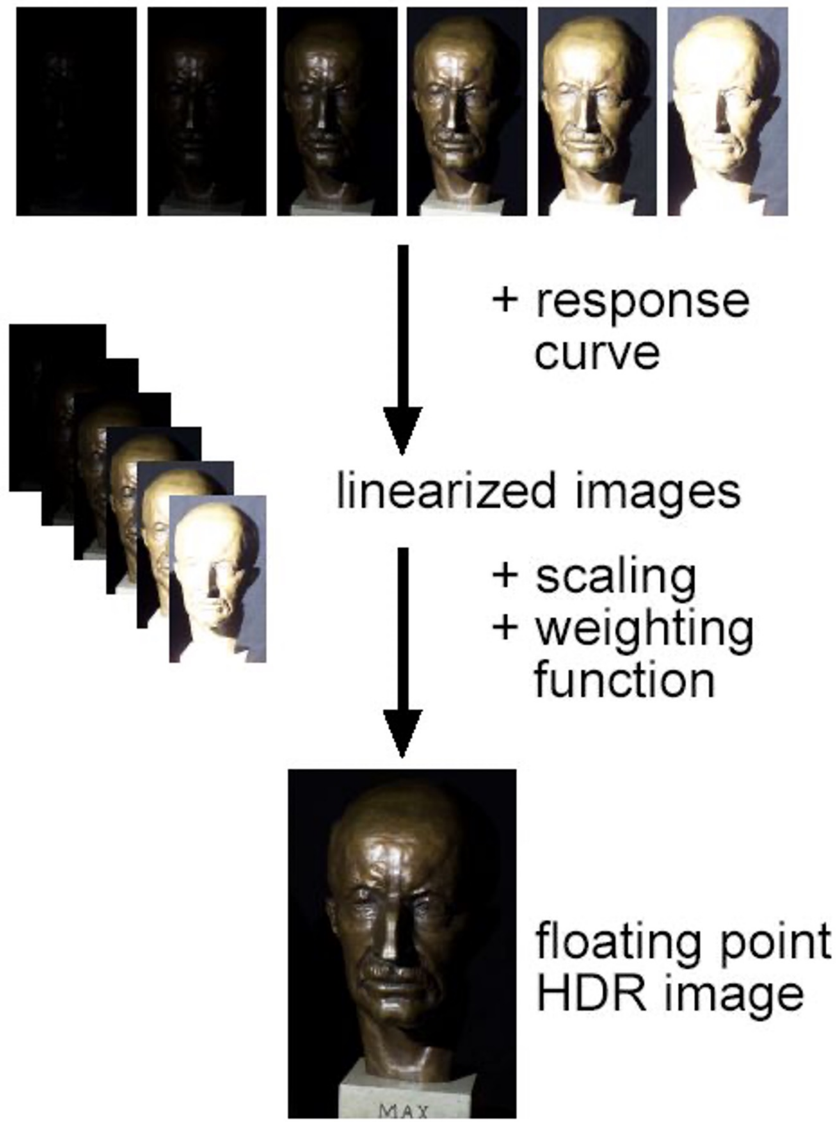
- **Similar situation for analog cameras**
  - Chemical development of film instead of electronic processing

  → Allows for varying sensitivity

# Multi-Exposure Techniques

- **Analog film**
  - Several emulsions of different sensitivity levels [Wyckoff 1960s]
    - Dynamic range of about $10^8$
- **Digital domain**
  - Similar approaches for digital photography
  - Commonly used method [Debevec et al. 97]
    - Select a small number of pixels from all images
    - Perform optimization of response curve with smoothness constraint
  - Newer method by [Robertson et al. 99]
    - Optimization over all pixels in all images
- **General idea of HDR imaging**
  - Combine multiple images with different exposure times
    - Pick for each pixel a well-exposed image
    - Response curve needs to be known to calibrate values betw. images
    - Change only exposure time, not aperture due to diff. depth-of-field !!

# Multi-Exposure Techniques



# HDR Imaging [Robertson et al. 99]

- **Principle of the approach**
  - Calculate an HDR image using the given response curve
  - Optimize response curve to better match resulting HDR image
  - Iterate till convergence: approx non-linear process w/ linear steps
- **Input**
  - Series of images *i* with exposure times $t_i$ and pixels *j*
  - Energy deposited at pixel *j* of image *i*: $I_{ij}$
  - Response curve *f* applied to incident energy yields pixel values $y_{ij}$

$$y_{ij} = f(I_{ij}) = f(t_i x_j)$$

- **Task**
  - Recover response curve: $f^{-1}(y_{ij}) = I_{ij}$
  - Determine irradiance $x_j$ at pixel *j* from energies $I_{ij}$: $x_j = I_{ij} / t_i$

# HDR Imaging [Robertson et al. 99]

- **Calculate estimates of HDR input values $x_j$ from images via maximum-likelihood approach**

$$x_j = \frac{\sum_i w_{ij} t_i^2 x_{ij}}{\sum_i w_{ij} t_i^2} = \frac{\sum_i w_{ij} t_i I_{ij}}{\sum_i w_{ij} t_i^2}$$

- **Use a bell-shaped weighting function $w_{ij} = w(y_{ij})$**
  - Do not trust as much pixel values at extremes
    - Under-exposed: high relative error prone to noise
    - Over-exposed: saturated value
- **Use an initial camera response curve**
  - Simple assumption: linear response

# HDR Imaging [Robertson et al. 99]

- **Optimizing the response curve $I(y_{ij})$**
  - Minimization of objective function $O$ (sum of weighted errors)

$$O = \sum_{i,j} w_{ij} \left(I_{ij} - t_i x_j\right)^2$$

  - Using standard Gauss-Seidel relaxation yields

$$I_m = \frac{1}{Card(E_m)} \sum_{i,j \in E_m} t_i x_j$$

$$E_m = \{(i,j) : y_{ij} = m\}$$

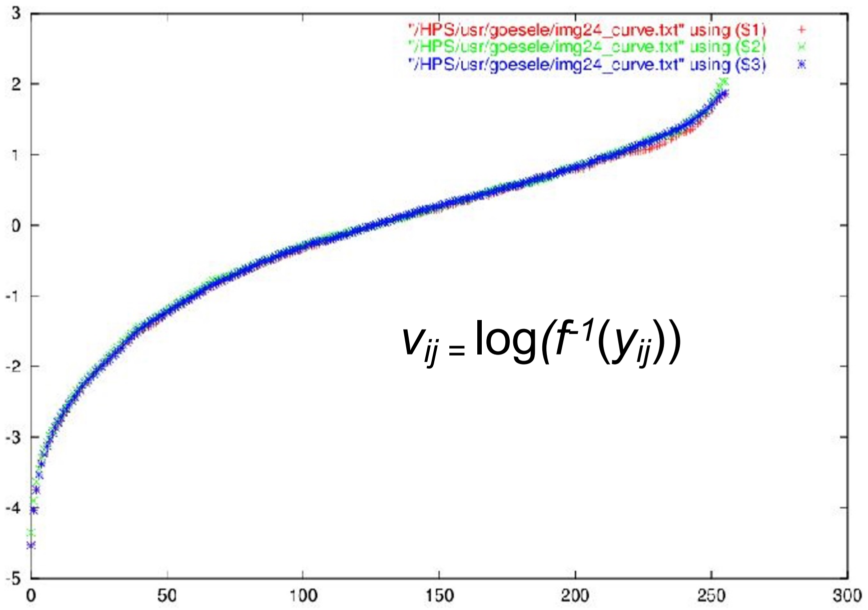
  - Normalization of $I$ so that $I_{128} = 1$

# HDR Imaging [Robertson et al. 99]

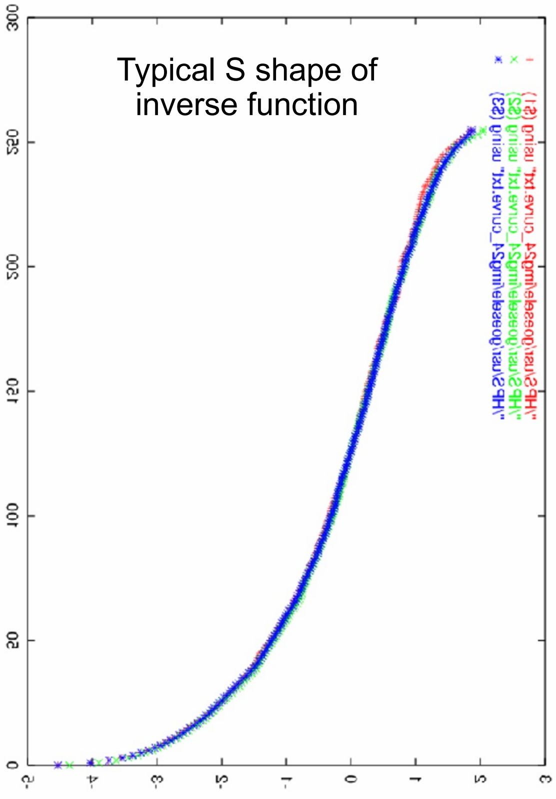
- **Both steps ...**
  - Calculation of an HDR image using *I*
  - Optimization of *I* using the HDR image

  **… are now iterated until convergence**
  - Criterion: decrease of *O* below some threshold
    - Usually about 5 iterations are enough
- **Logarithmic plot of the response curve**

$$y_{ij} = f(\exp(v_{ij}))$$




$$I_{ij} = \exp(v_{ij})$$
$$v_{ij} = \log(I_{ij})$$



# Choice of Weighting Function

- ***w***(***y***$_{ij}$) **for response [Robertson et al. 99]**

$$\mathrm{w}_{ij} = \exp\left(-4\frac{\left(y_{ij} - 127.5\right)^2}{127.5^2}\right)$$

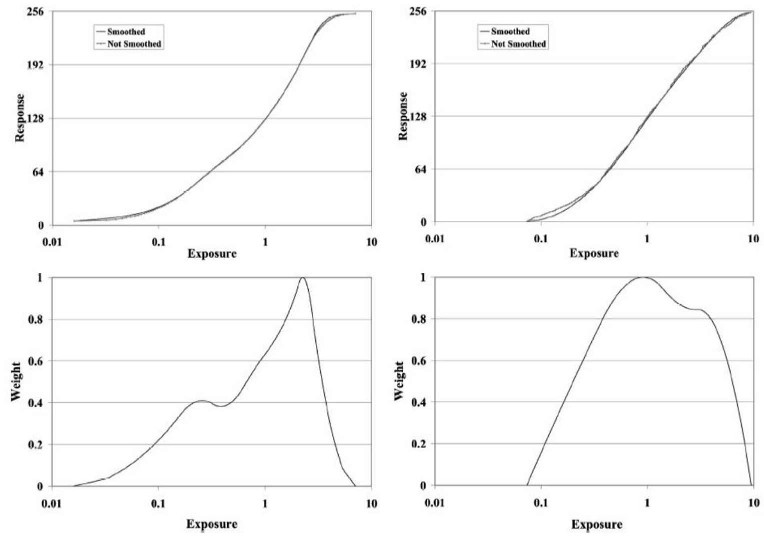
  - Gaussian-like bell-shaped function
  - For 8-bit images, centered around $(2^8 - 1) / 2 = 127.5$
  - Possible width correction at both ends: over/under-exposure
  - Motivated by general noise model: downweight high relative error

- ***w***(***y***$_{ij}$) **for HDR reconstruction [Robertson et al. 03]**
  - Introduce certainty function *c* as derivative of response curve with logarithmic exposure axis: S-shape response→bell-shaped curve
  - Approxim. response curve with cubic spline to compute derivative

$$w_{ij} = w(y_{ij}) = c(I_{ij})$$

# Weighting Function

- **Consider response curve gradient**
  - Higher weight where response curve maps to large extent




[Robertson et al. 2003]

- **Difference between exposures levels**
  - Ideally such that respective trusted regions (central part of weighting function) are roughly adjacent

# HDR Generation

- **What difference to pick between exposures levels?**
  - Most often a difference of 2 stops (factor of 4) between exposures is sufficient
  - See [Grossberg & Nayar 2003] for more details
- **How many input images are necessary to get good results?**
  - Depends on dynamic range of scene illumination and on quality requirements
  - Often 3 images are fine (normal + lower & higher exposure)

# Algorithm of Robertson et al.

- **Discussion**
  - Method is very easy
  - Doesn't make assumptions about response curve shape
  - Converges quickly
  - Takes all available input data into account
    - As opposed to [Debevec et al. 97]
  - Can be extended to > 8-bit color depth
    - 16 bits should be followed by smoothing
    - Quantization to 8 bits eliminates large amount of noise
    - Higher precision with 16 bits more likely to still contain notable noise

Part II

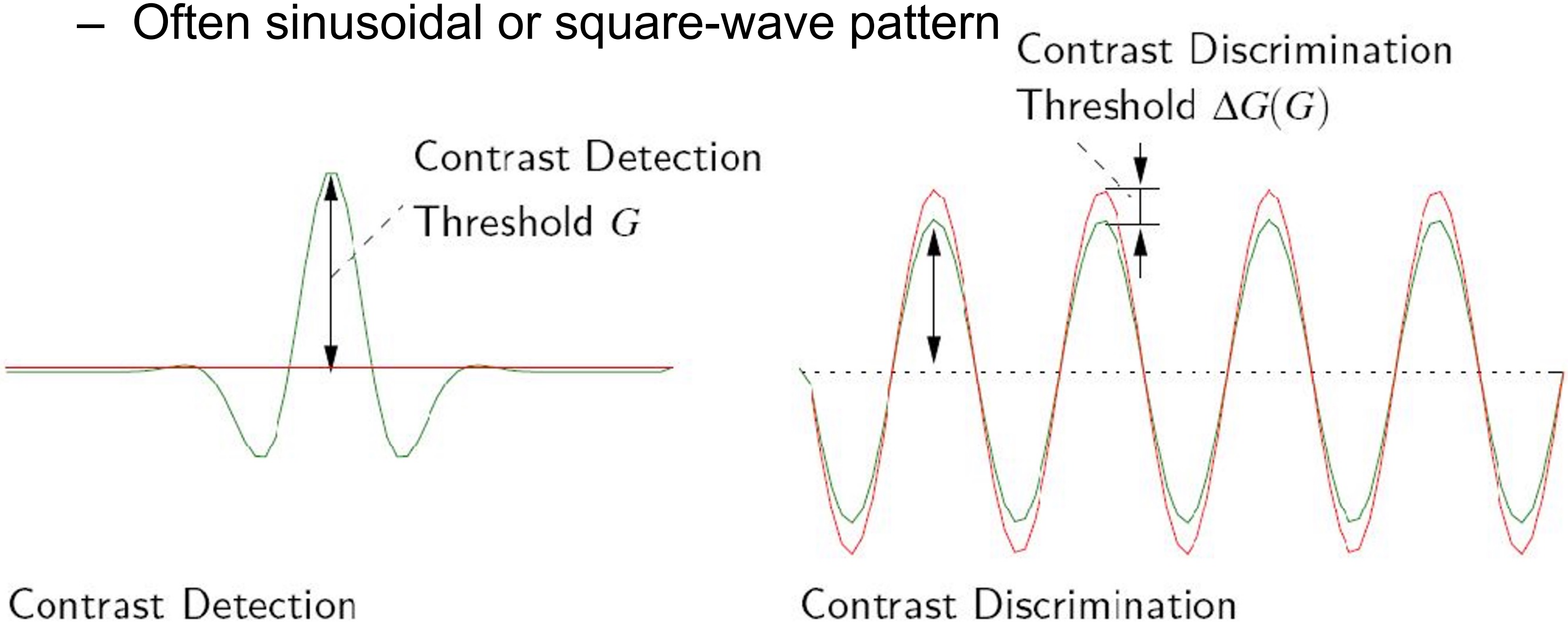
# Tone Mapping

# Terms and Definitions

- **Dynamic range**
  - Factor between the highest and the smallest representable value
  - 2 strategies to increase dynamic range:
    - Make white brighter, or make black darker (more practical)
    - Reason for trend towards reflective rather than diffuse displays
- **Contrast**
  - Simple contrast: $C_S = \frac{L_{max}}{L_{min}}$
  - Weber fraction: $C_W = \frac{\Delta L}{L_{min}}$ with $\Delta L = L_{max} - L_{min}$
  - Michelson contrast: $C_M = \frac{|L_{max} - L_{min}|}{L_{max} + L_{min}}$
  - Logarithmic ratio: $C_L = \log_{10}\left(\frac{L_{max}}{L_{min}}\right)$
  - Signal to noise ratio (SNR): $C_{SNR} = 20 \cdot \log_{10}\left(\frac{L_{max}}{L_{min}}\right)$

# Contrast Measurement

- **Contrast detection threshold**
  - Smallest detectable intensity difference in a uniform field of view
  - E.g., Weber-Fechner perceptual experiments
- **Contrast discrimination threshold**
  - Smallest visible difference between two similar signals
  - Works in supra-detection-threshold domain (i.e., signals above it)
  - Often sinusoidal or square-wave pattern



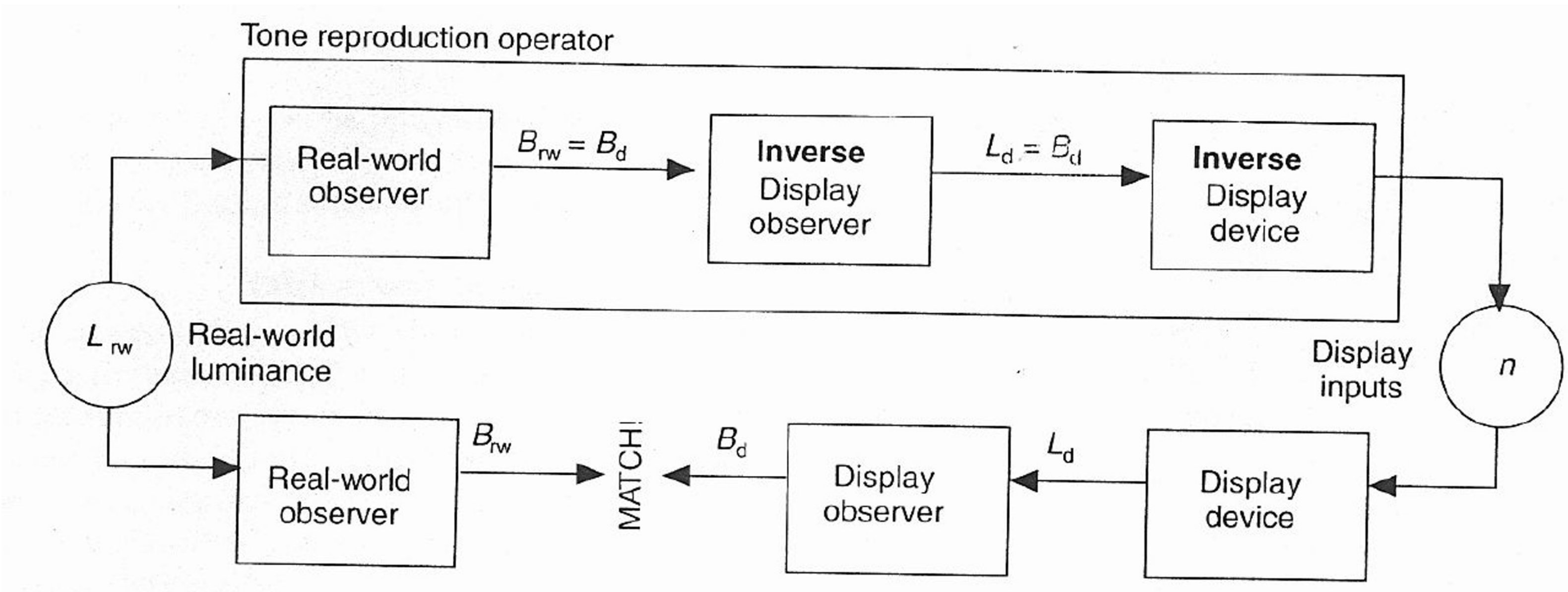
# Why Tone Mapping?

- **Mapping HDR radiance values to LDR pixel values?**
  - Luminance range for human visual perception
    - Min $10^{-5}$ cd/m$^2$: shadows under starlight
    - Max $10^5$ cd/m$^2$ : snow in direct sunlight
  - Luminance of typical desktop displays
    - Typically, up to ~500 cd/m$^2$ (some now >1000 cd/m$^2$)
      - About 2-3 orders of magnitude
      - Darkest black determined by technology & reflection of screen

- **Goal**
  - Compress the dynamic range of an input image to fit output range
  - Reproduce HVS to closely match perception of the real scene
    - Brightness and contrast
    - Adaptation of the eye to environment
    - Bright/dark input: glare, color perception, loss of visual acuity, …

# General Principle

- **Original approach [Tumblin/Rushmeier]**
  - Create model of the observer
  - Requirement: observer looking at displayed virtual image should perceive the same brightness as when staring at the real scene
  - Compute tone-mapping as concatenation/inversion of operators
  - Model usually operates only on luminance (not on color)




- **Other models aim for visually pleasing images**

# Heuristic Approaches

- **Linearly scale brightest value to 1 (in gray value)**
  - Problem: light sources are often several orders of magnitude brighter than the rest → the rest will be black
- **Linearly scale brightest non-light-source value**
  - Capping light source values to 1
  - Scale the rest to a value slightly below 1
  - Problem: bright reflections of light sources

- **General problem of simple linear scaling**
  - Absolute brightness gets lost
  - Scaling of light source intensity gets factored out → has no effect
- **Much better: linear scaling in the logarithmic domain**
  - Linear scaling of perceived brightness instead of input luminance
  - Much closer to human perception
  - Typically using $\log_{10}$

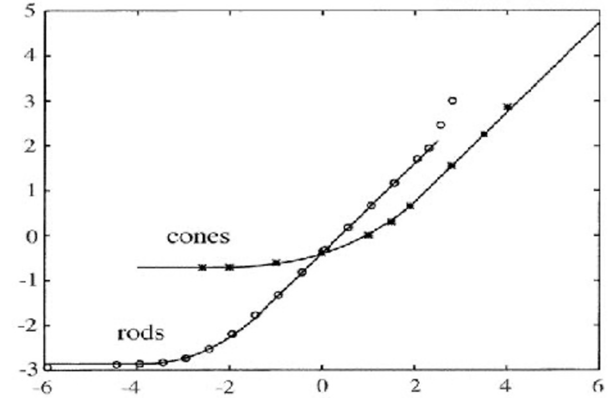
# Maintaining Contrast

- **Contrast-based linear scaling factor [Ward 94]**
  - Make just visible differences in real world just visible on display
    - Preserve the visibility in the scene based on Weber's contrast
  - Just noticeable contrast differences according to Blackwell [CIE 81] (subjective measurements)

$$\Delta L(L_a) = 0.0594(1.219 + L_a^{0.4})^{2.5}$$

    - Minimum discernible difference in luminance for given visual adaptation level $L_a$

Threshold $\Delta L$ [log cd/m²]




Adaptation luminance $L_a$ [log cd/m²]

  - Goal: proportionality constant $m$
    - Relates world luminance values $L_w$ to display luminance values $L_d$
    - $L_d = m L_w$

# Maintaining Contrast

- **Approach using "just noticeable difference" (JND)**
  - Find m such that JND $\Delta L(L_{wa})$ at world adaptation luminance $L_{wa}$ and JND $\Delta L(L_{da})$ at display adaptation luminance $L_{da}$ verify

$$\Delta L(L_{da}) = m(L_{wa}) \Delta L(L_{wa})$$

  - Substitution results in

$$m(L_{wa}) = \left[ \frac{1.219 + L_{da}^{0.4}}{1.219 + L_{wa}^{0.4}} \right]^{2.5}$$

  - Compute $L_{da}$ from maximum display luminance: $L_{da} = L_{dmax} / 2$
  - Normalize scaling factor *sf* in [0, 1]

$$sf = \frac{1}{L_{dmax}} \left[ \frac{1.219 + (L_{dmax}/2)^{0.4}}{1.219 + L_{wa}^{0.4}} \right]^{2.5}$$

# Maintaining Contrast

- **Deriving the real-world adaptation $L_{wa}$**
  - Depends on light distribution in field of view of observer
  - Simple approximation using a single value
    - Eyes try to adjust to average incoming brightness
    - Brightness *B* based on input luminances:
      - $B = k\, L_{in}^{a}$ : Power-law [Stevens 61]
    - Comfortable brightness based on average of input luminances:
      - $\log_{10}(L_{wa}) = E\{\log_{10}(L_{in})\} + 0.84 \Rightarrow L_{wa} = 10^{\wedge}(\sum_n \log_{10}(L_{in}) / n)$
- **Problems of this approach**
  - Single factor for entire image
    - Does not handle different adaptation for different locations in image
    - We do not perceive absolute differences in luminance: neighborhood
  - Brightness adaptation mainly acts on 1° field of view of fovea rather than periphery → would require eye tracking
  - Adaptation to average results in clamping for too bright regions

# Histogram Adjustment

- **Optimal mapping of the dynamic range [Ward 97]**
  - Compute an adjustment image
    - Assume known viewpoint with respect to the scene
    - Blur input image with distance-dependent kernel
      - Filter (average) non-overlapping regions covering 1° field of view, i.e., foveal solid angle of adaptation
      - Reference uses simple box filter
    - Reduce resolution
  - Compute the histogram of the image
    - Bin the luminance values
  - Adjust the histogram based on restrictions of HVS
    - Limit contrast enhancement

$\Rightarrow$ **Distributes contrast in the image in a visually meaningful way, but does not try to model human vision per se as outlined by [Tumblin/Rushmeier]**

# Histogram Adjustment

- **Definitions**
  - $B_w = \log(L_w)$ : compute world brightness from world luminance
  - $b_i$ : create $N$ bins $i$ corresponding to ranges of $B_w$
  - $f(b_i)$ : number of $B_w$ samples in bin $b_i$: $\propto$ PDF
  - $P(b)=\Sigma f(b_i)/T$ : normalized sum of $f(b_i)$ for $b_i < b$: CDF ($\int$ of PDF)
  - $T$ : sum over all $f(b_i)$, i.e., total number of samples

$$T = \sum f(b_i)$$

$$\Delta b = \frac{\log(L_{w}max) - \log(L_{w}min)}{N}$$

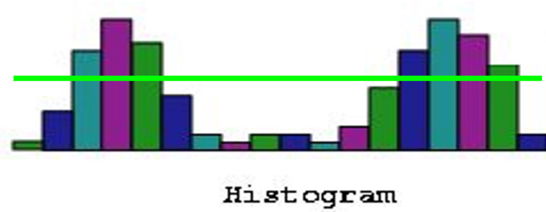
  - Bin step size $\Delta b$ (in log(cd/m$^2$)) defined by min/max world luminance for the scene and number of histogram bins $N$
  - Therefore, the PDF is

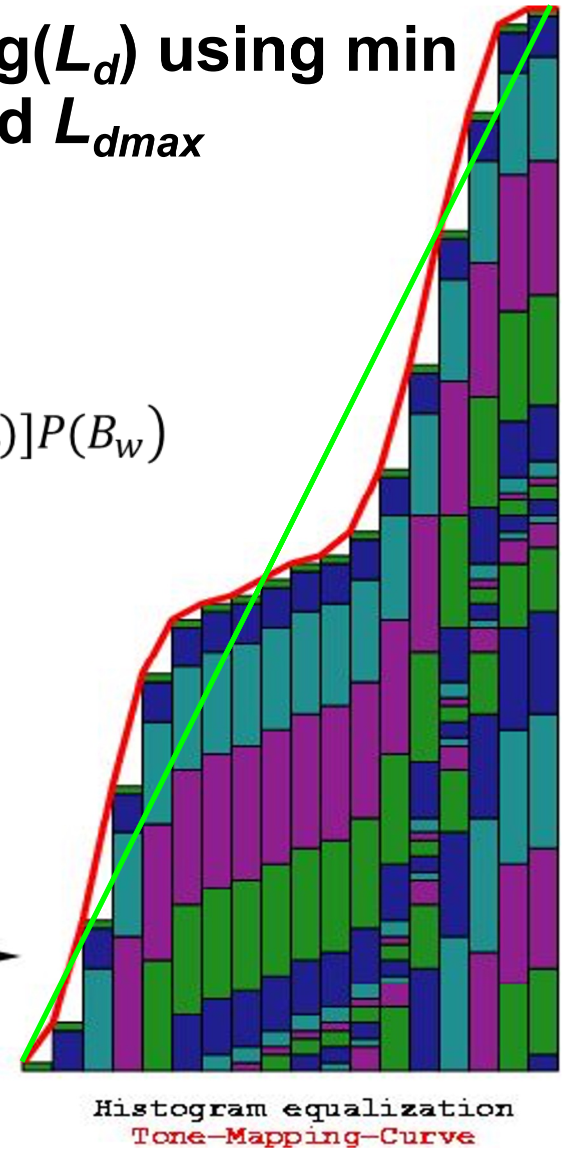
$$dP(b) / db = f(b_i) / (T \Delta b)$$

# Naïve Histogram Equalization

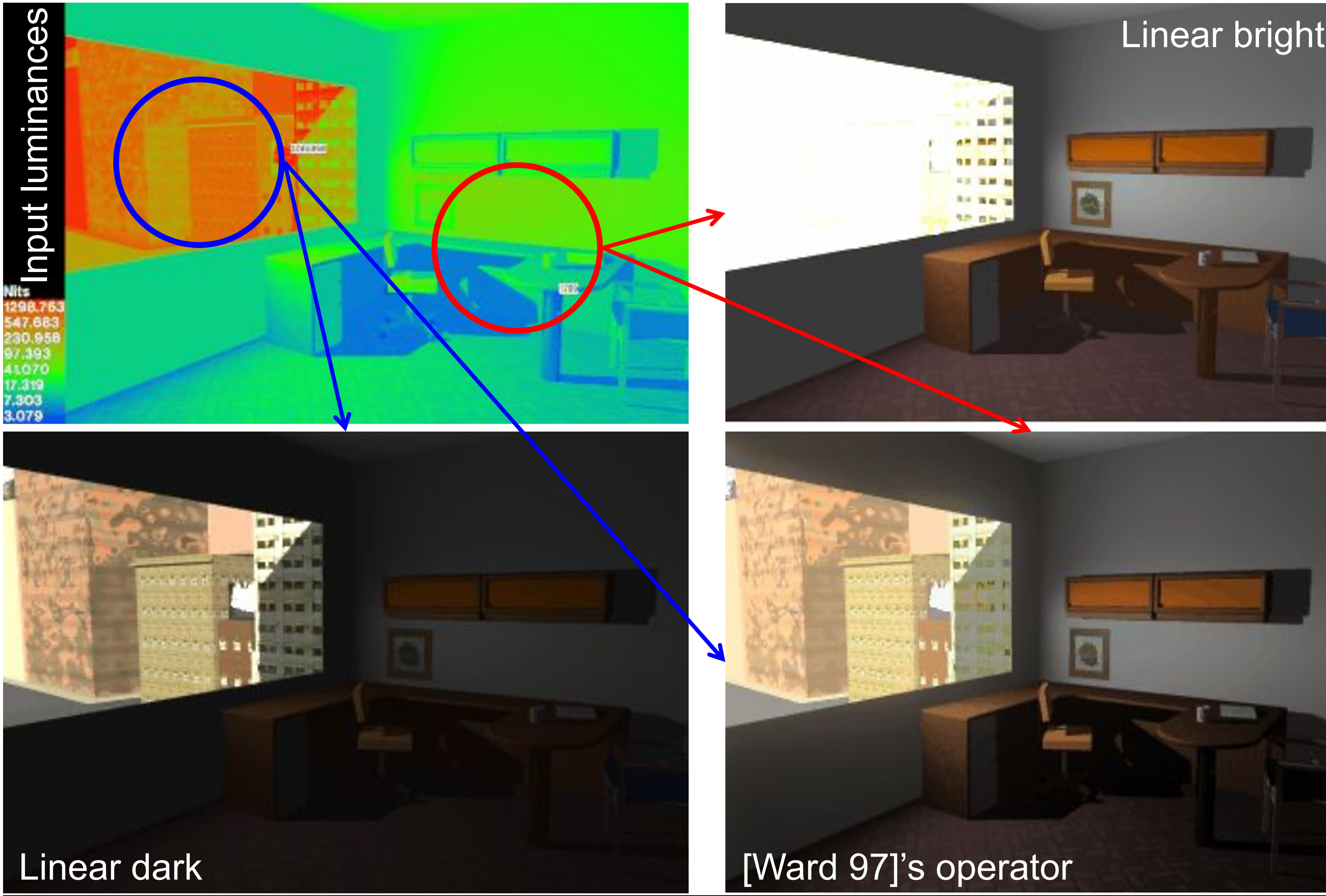
- **Compute display brightness $B_d = \log(L_d)$ using min and max display luminance $L_{dmin}$ and $L_{dmax}$**

$$B_d = \log(L_{dmin}) + [\log(L_{dmax}) - \log(L_{dmin})]P(B_w)$$



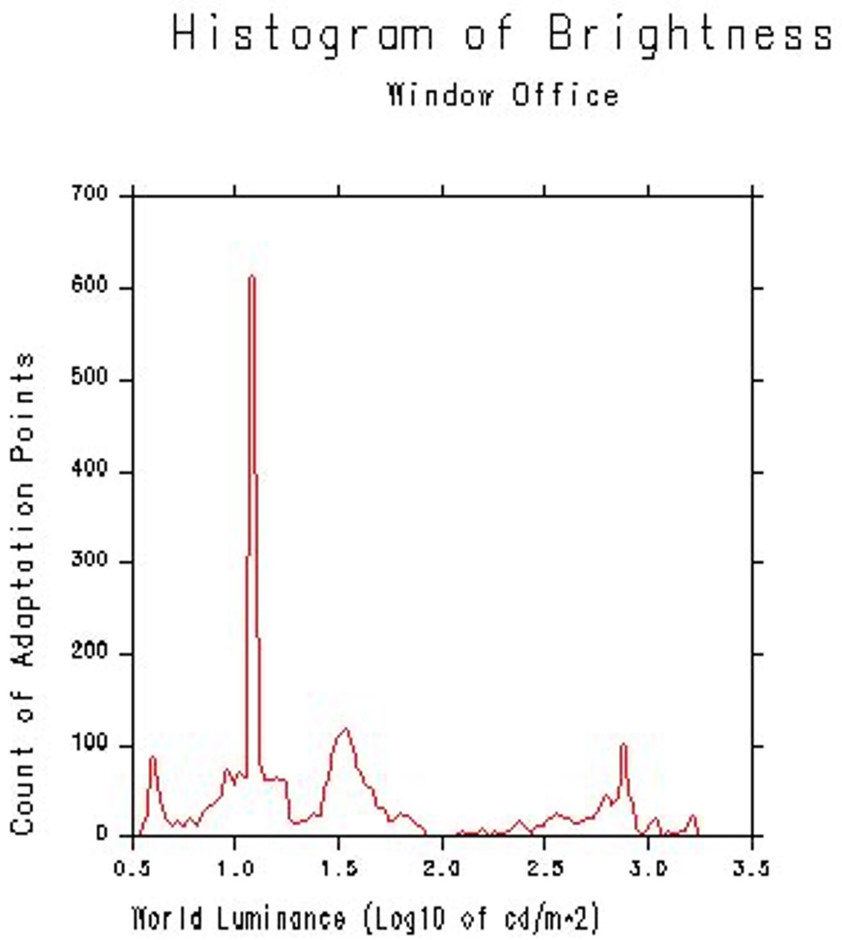



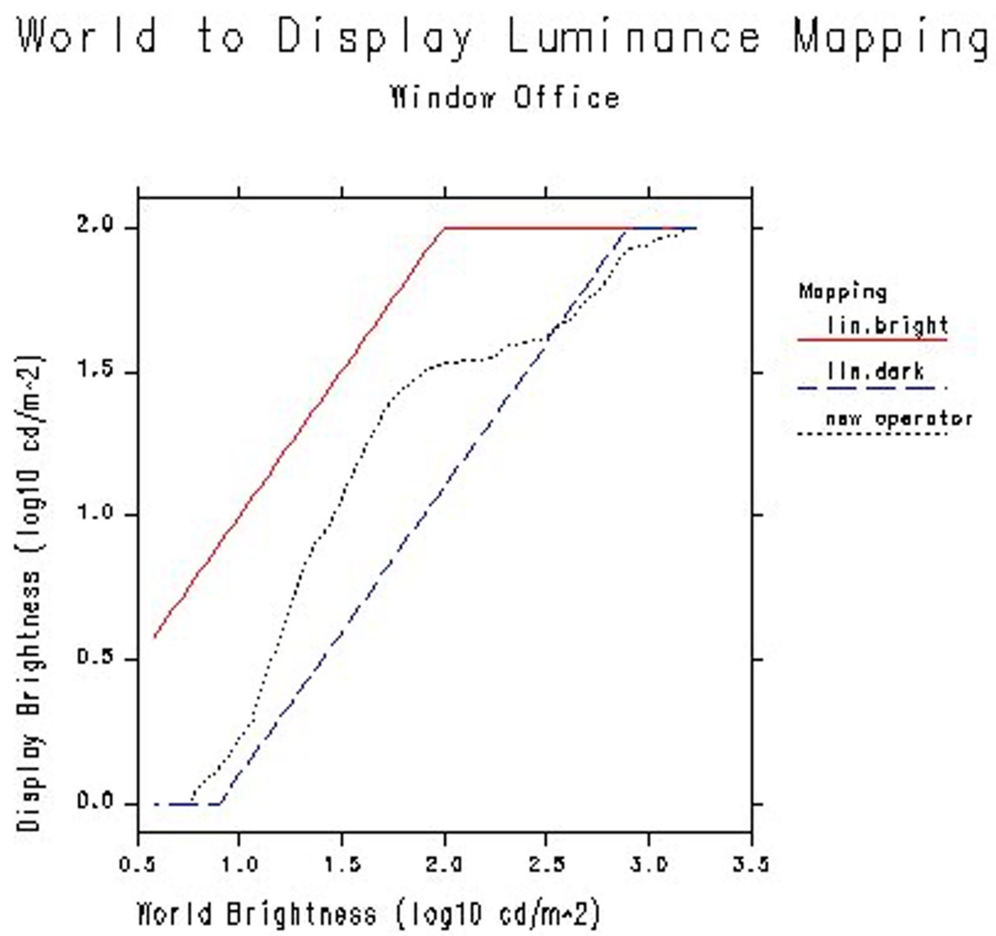
# Histogram Adjustment



# Histogram Adjustment

- **Linear mapping (scaling) vs. histogram adjustment**



# Histogram Adj. w/ Linear Ceiling

- **Problem**
  - Too exaggerated contrast in large highly-populated regions of the dynamic range: enhances features more than the HVS would
- **Idea**
  - Contrast-limited histogram equalization using a linear ceiling (linear scaling works well for low contrast images)

$$\frac{dL_d}{L_d} \leq \frac{dL_w}{L_w} \Rightarrow \frac{dL_d}{dL_w} \leq \frac{L_d}{L_w}$$

  - Differentiate $L_d = \exp(B_d)$ with respect to $L_w$ using the chain rule

$$\frac{dL_d}{dL_w} = \exp(B_d) \frac{f(B_w)}{T\Delta b} \frac{\log(L_{dmax}) - \log(L_{dmin})}{L_w} \leq \frac{L_d}{L_w}$$

- **Result**
  - Limiting the sample count per bin in the histogram

  ⇔ limit the magnitude of the PDF, i.e., the slope of the CDF

$$f(B_w) \leq \frac{T\Delta b}{\log(L_{dmax}) - \log(L_{dmin})}$$

# Histogram Adj. w/ Linear Ceiling

- **Implementing the contrast limitation**
  - Truncate too large bins w/ redistribution to neighbors (repeatedly)
  - Ditto without redistribution (gives better results)
  - Use modified $f(B_w)$ in histogram equalization vs. naïve approach

# Histogram Adj. w/ Linear Ceiling

Linear mapping (simple scaling)

Naïve histogram equalization

Histogram adjustment with linear ceiling on contrast

# HA based on Hum. Contr. Sensi.

- **Adjustment for JND**
  - Limiting the contrast to the ratio of JNDs (global scale factor)

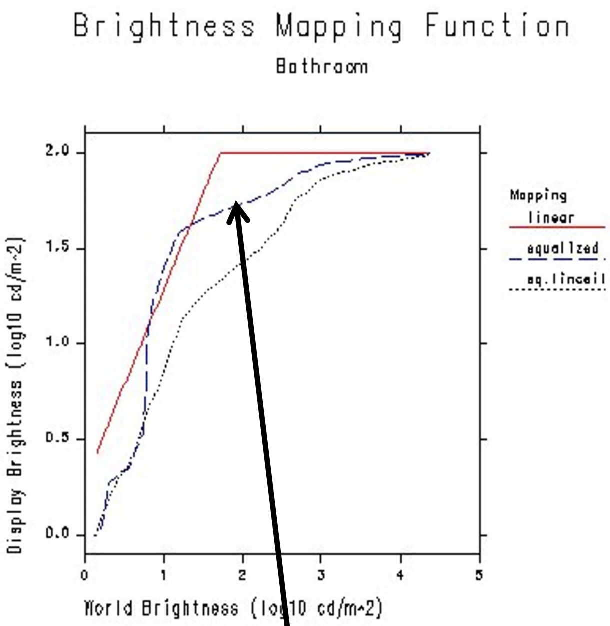
$$\frac{dL_d}{dL_w} \leq \frac{\Delta L_t(L_d)}{\Delta L_t(L_w)}$$

  - That results in

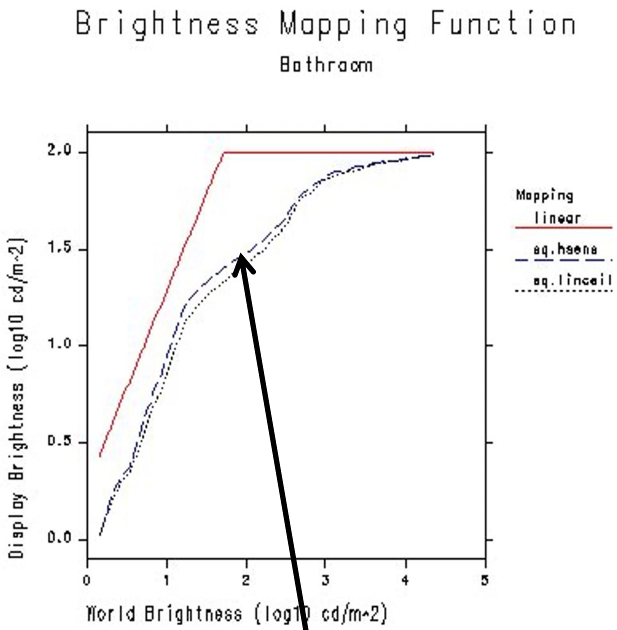
$$f(B_w) \leq \frac{\Delta L_t(L_d)}{\Delta L_t(L_w)} \frac{L_w}{L_d} \frac{T \Delta b}{[\log(L_{d\,max}) - \log(L_{d\,min})]}$$

  - Implementation is similar as for previous histogram equalization

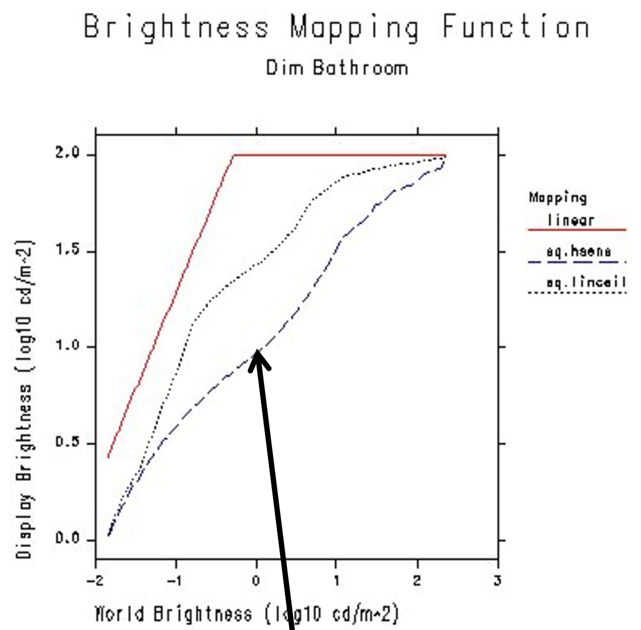
# HA based on Hum. Contr. Sensi.




Naïve histogram equalization




HA with human sensitivity in bright bathroom




HA with human sensitivity in dim bathroom

# HA based on Hum. Contr. Sensi.

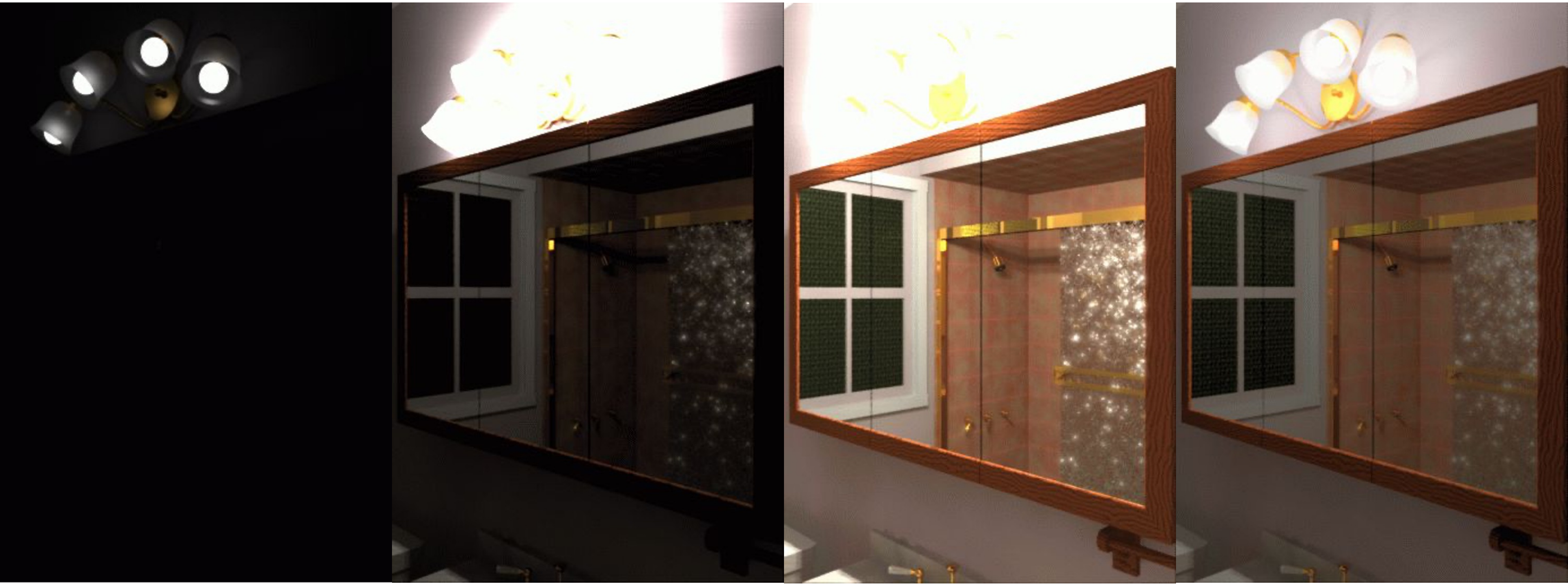
- **Reduction of contrast sensitivity in dark scenes**

Histogram adjustment with linear ceiling on contrast

Dim bathroom (1/100) with reduced contrast

# Comparison

- **[Tumblin/Rushmeier]**
  - Sound methodology from a theoretical standpoint
  - Maybe not optimal models of HVS used in practical experiments

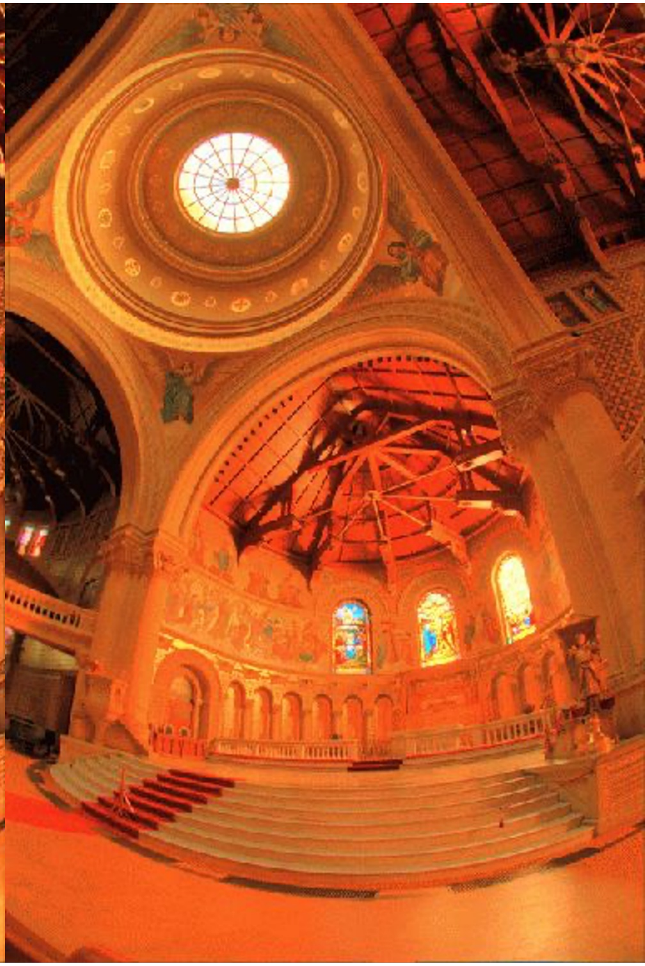


**Maximum linear scaling tone mapping** | **[Tumblin/Rushmeier] tone mapping** | **Contrast-based lin. scal. [Ward 94] tone mapping** | **Histogram adjustment [Ward 97] tone mapping**

# Comparison

**[Tumblin/Rushmeier] tone mapping**

**Contrast-based linear scaling [Ward 94] tone mapping**



**Histogram adjustment [Ward 97] tone mapping**