Computer Graphics

- Rasterization -

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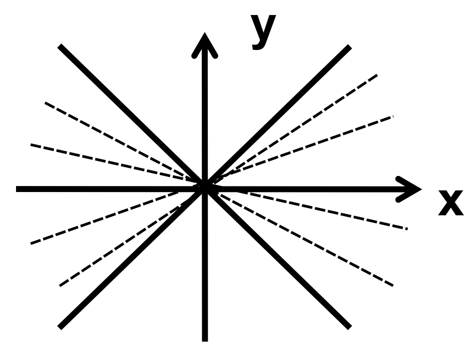
Rasterization

- **Definition**
  - Given some 2D geometry (point, line, circle, triangle, polygon,…), specify which pixels of a raster display each primitive *covers*
    - Sometimes also called "scan-conversion" (for historic reasons)
  - Anti-aliasing: Instead of only fully-covered pixels (single sample), specify what parts of a pixel are *covered* (multi/super-sampling)
- **Perspectives**
  - OpenGL lecture: from an application programmer's point of view
  - This lecture: from a graphics package implementer's point of view
  - Looking at rasterization of (i) lines and (ii) polygons (areas)
- **Usages of rasterization in practice**
  - 2D-raster graphics, e.g. Postscript, PDF, SVG, …
  - 3D-raster graphics, e.g. SW rasterizers (Mesa, OpenSWR), HW
  - 3D volume modeling and rendering
  - Volume operations (CSG operations, collision detection)
  - Space subdivision (spatial indices): Construction and traversal

Rasterization

- **Assumptions**
  - Pixels are sample **points** on a 2D integer grid
    - OpenGL: at cell bottom-left, integer-coordinate
    - X11, Foley: at the cell center (we will use this)
  - Simple raster operations
    - Just setting pixel values or not (binary decision)
    - More complex operations later: compositing/anti-aliasing
  - Endpoints snapped to (sub-)pixel integer coordinates
    - Simple and consistent computations with fixed-point arithmetic
  - Limiting to lines with gradient/slope $|m| \leq 1$ (mostly horizontal)
    - Separate handling of horizontal and vertical lines
    - For mostly vertical, swap x and y ($|1/m| \leq 1$), rasterize, swap back
      - Special cases in SW, trivial in HW :-)
  - Line width is one pixel
    - $|m| \leq 1$: 1 pixel per column (X-driving axis)
    - $|m| > 1$: 1 pixel per row (Y-driving axis)



Lines: As Functions

- **Specification**
  - Initial and end points: $(x_b, y_b), (x_e, y_e), (dx, dy) = (x_e - x_b, y_e - y_b)$
  - Functional form: $y = mx + B$
  - End points with integer coordinates $\Rightarrow$ rational slope $m = dy/dx$
- **Goal**
  - Find that pixel per column whose distance to the line is smallest
- **Brute-force algorithm**
  - Assume that +X is the driving axis → set pixel in every column

```
for x_i = x_b to x_e
    y_i = m * x_i + B
    setPixel(x_i, Round(y_i))     // Round(y_i) = Floor(y_i + 0.5)
```

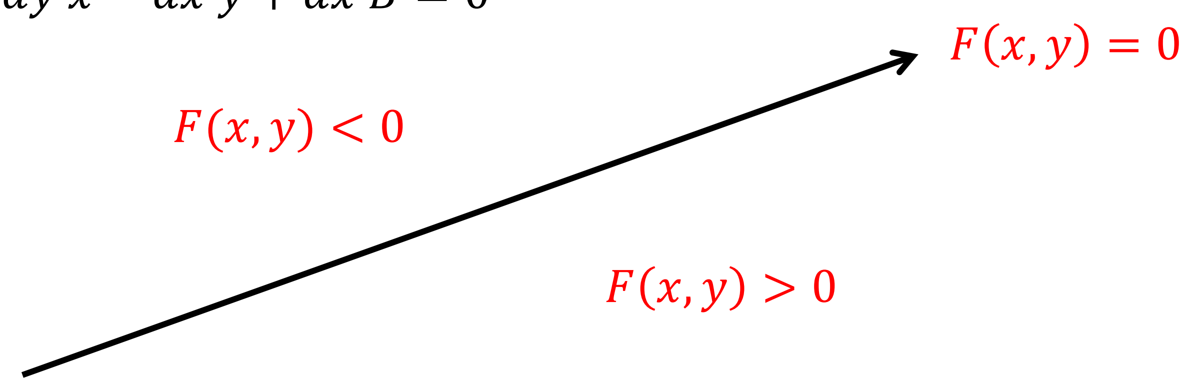
- **Comments**
  - Variables *m* and thus y_i need to be calculated in floating-point
  - Not well suited for direct HW implementation
    - A floating-point ALU is significantly larger in HW than integer

Lines: DDA

- **DDA: Digital Differential Analyzer**
  - Origin of incremental solvers for simple differential equations
    - E.g. Euler method
  - Per time-step: $x' = x + dx/dt$, $y' = y + dy/dt$
- **Incremental algorithm**
  - Choose dt=dx, then per pixel
    - $x_{i+1} = x_i + 1$
    - $y_{i+1} = m * x_{i+1} + B = m(x_i + 1) + B = (m * x_i + B) + m = y_i + m$
    - setPixel(x_{i+1}, Round(y_{i+1}))
- **Remark**
  - Utilization of coherence through incremental calculation
    - Avoids the “costly” multiplication
  - Accumulates error over length of the line
    - Up to 4k additions on UHD!
  - Floating point calculations may be moved to fixed point
    - Must control accuracy of fixed-point representation
    - Enough extra bits to hide accumulated error (>>12 bits for UHD)

Lines: Bresenham (1963)

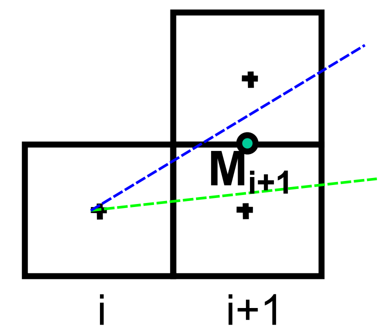
- **DDA analysis**
  - Critical point: decision whether we need rounding up or down
- **Idea**
  - Integer-based decision through implicit functions
  - Implicit line equation (similar to Window Edge Coordinates)
    - $F(x, y) = ax + by + c = 0$
  - Here with $y = mx + B = \frac{dy}{dx}x + B \quad \Rightarrow \quad 0 = dy\ x - dx\ y + B\ dx$
    - $\boldsymbol{a = dy, \quad b = -dx, \quad c = Bdx}$
  - Results in
    - $F(x, y) = dy\ x - dx\ y + dx\ B = 0$



Lines: Bresenham

- **Decision variable *d* (the midpoint formulation)**
  - Assume we are at x=i, calculating next step at x=i+1
  - Measures the vertical distance of midpoint from line:

$$\begin{aligned} d_{i+1} &= F(M_{i+1}) = F(x_i + 1, y_i + 1/2) \\ &= a(x_i + 1) + b(y_i + 1/2) + c \end{aligned}$$



- **Preparations for the next pixel**

IF ($d_{i+1} \leq 0$) // Increment in x only

$d_{i+2} = d_{i+1} + a = d_{i+1} + dy$ // Incremental calculation

ELSE // Increment in x and y

$d_{i+2} = d_{i+1} + a + b = d_{i+1} + dy - dx$

y = y + 1

ENDIF

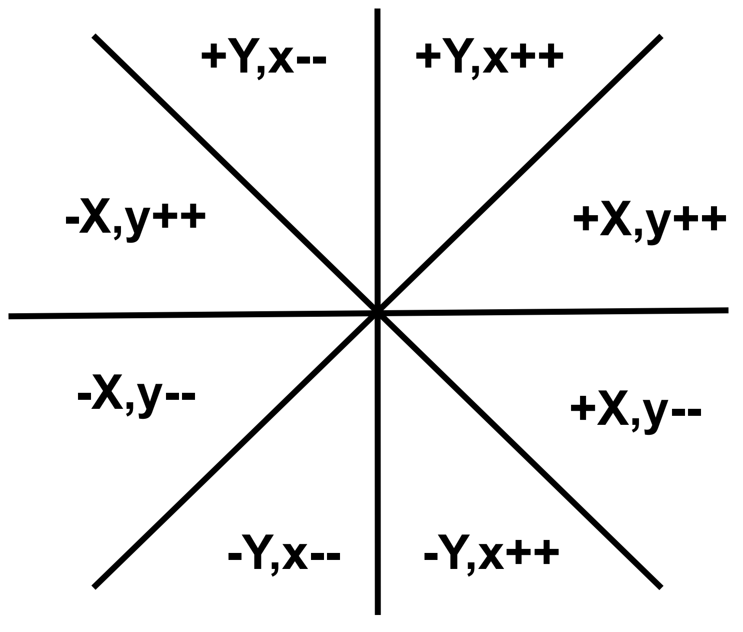
x = x + 1

Lines: Integer Bresenham

- **Initialization**
  - $d_1 = F\left(x_b + 1, y_b + \frac{1}{2}\right) = a(x_b + 1) + b\left(y_b + \frac{1}{2}\right) + c$
    $= ax_b + by_b + c + a + \frac{b}{2} = F(x_b, y_b) + a + \frac{b}{2} = a + \frac{b}{2}$
  - Because $\mathrm{F}(x_b, y_b)$ is zero by definition (line goes through (x_b, y_b))
    - Pixel is always set (but check consistency rules → later)
- **Elimination of fractions**
  - Any positive scale factor maintains the sign of $F(x,y)$
    - $2F(x_b, y_b) = 2(ax_b + by_b + c) \rightarrow d_{start} = 2a + b$
- **Observation:**
  - When the start and end points have integer coordinates then b = -dx and a = dy are also integers
    - Floating point computation can be eliminated
  - No accumulated error!!

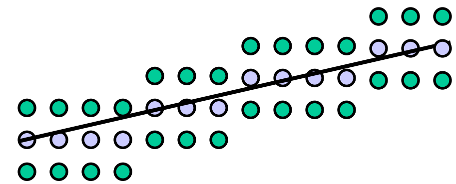
Lines: Arbitrary Directions

- **8 different cases**
  - Driving (active) axis: ±X or ±Y
  - Increment/decrement of *y* or *x*, respectively

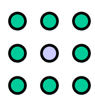


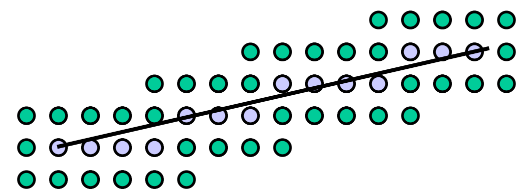
Thick Lines

- **Pixel replication**

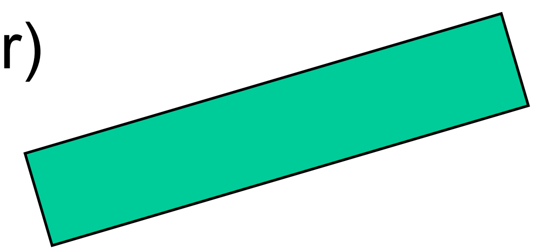


  - Problems with even-numbered widths
  - Varying intensity of a line as a function of slope



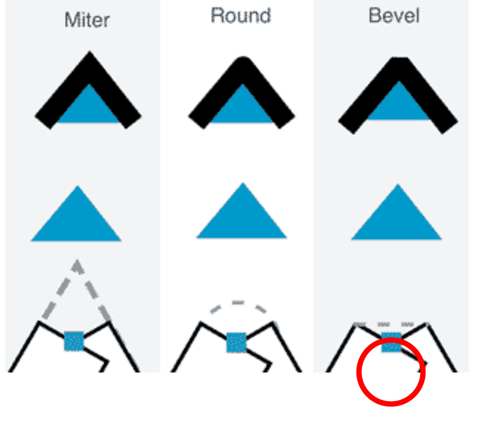


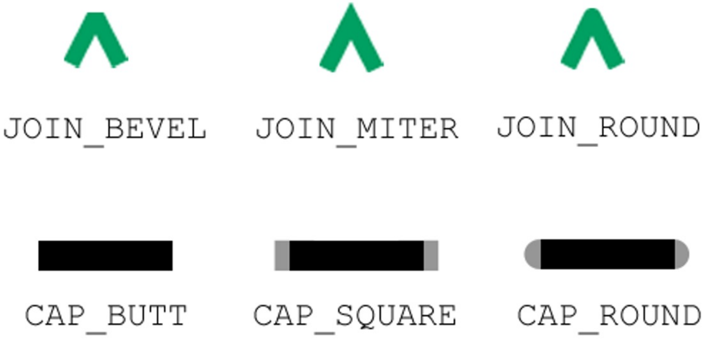
- **The moving pen**
  - For some pen footprints the thickness of a line might change as a function of its slope
  - Should be as “round” as possible
- **Real Solution: Draw 2D area** (see later)
  - Allows for anti-aliasing and fractional width
  - Main approach these days!



Handling Start and End Points

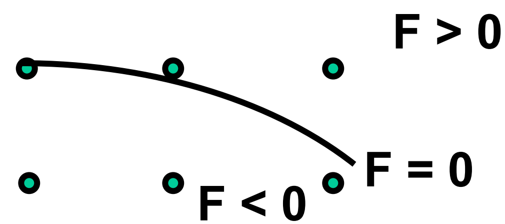
- **End points handling (not available in current OpenGL)**
  - Joining: handling of joints between lines
    - Bevel: connect outer edges by straight line
    - Miter: join by extending outer edges to intersection
    - Round: join with radius of half the line width
  - Capping: handling of end point
    - Butt: end line orthogonally at end point
    - Square: end line with oriented square
    - Round: end line with radius of half the line width
  - Avoid overdraw when lines join





Bresenham: Circle

- **Eight different cases, here +X, y--**



Initialization: x = 0, y = R

$F(x,y) = x^2+y^2-R^2$

d = F(x+1, y-1/2)

IF d < 0

    d = F(x+2,y-1/2)

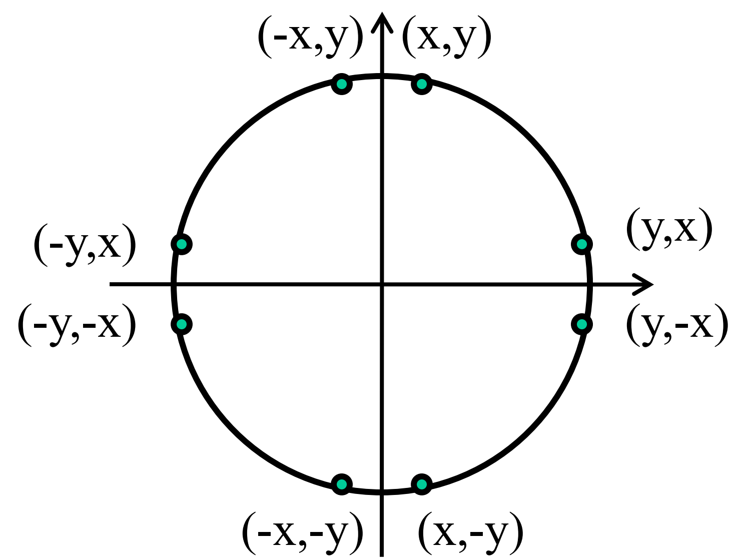
ELSE IF d > 0

    d = F(x+2,y-3/2)

    y = y-1

ENDIF

x = x+1

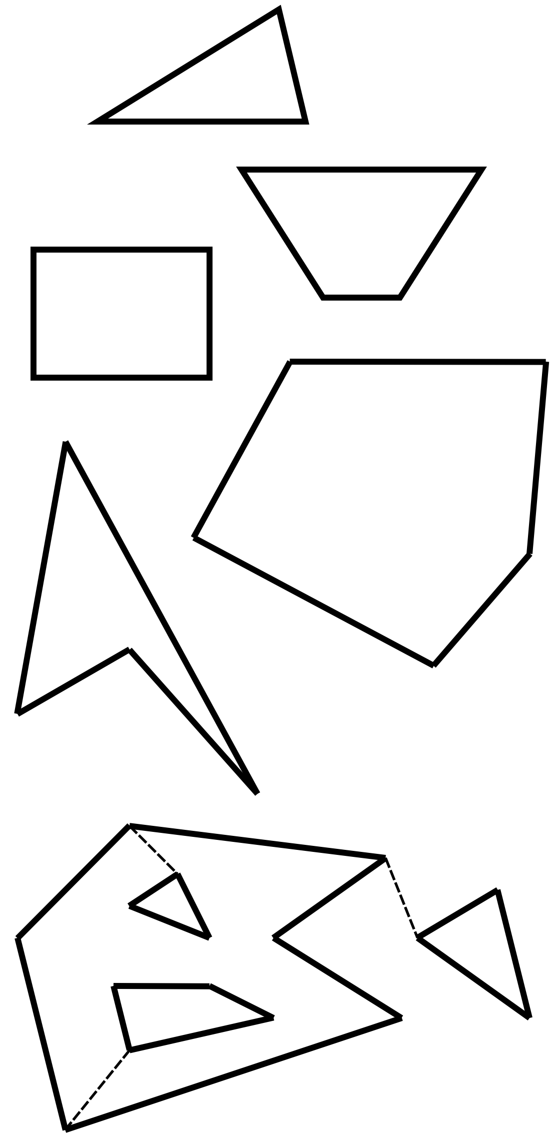


  – **Works because |slope| is smaller than 1**

- **Eight-way symmetry: only one 45° segment is needed to determine all pixels in a full circle**

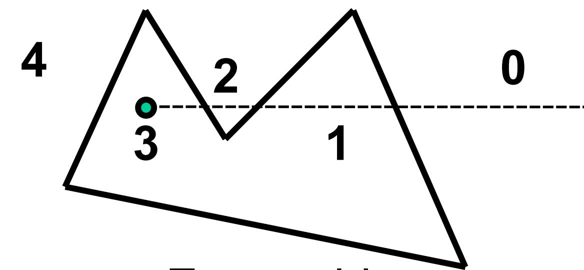
Reminder: Polygons

- **Types**
  - Triangles
  - Trapezoids
  - Rectangles
  - Convex polygons
  - Concave polygons
  - Arbitrary polygons
    - Holes
    - Overlapping
- **Two approaches**
  - Polygon tessellation into triangles
    - Only option for OpenGL
  - Direct scan-conversion
    - Mostly in early SW algorithms



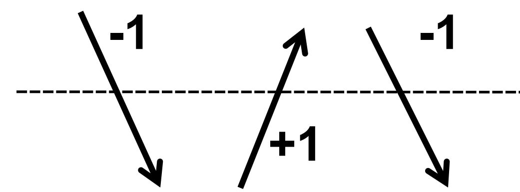
Inside-Outside Tests

- **What is the interior of a polygon?**
  - Jordan curve theorem
    - „Any continuous ***simple*** closed curve in the plane, separates the plane into two disjoint regions, the inside and the outside, one of which is bounded.“

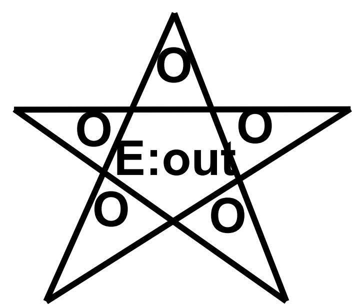


Even-odd

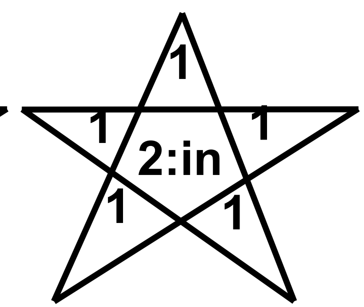
- **What to do with *non-simple* polygons?**
  - Even-odd Winding Number rule (odd parity)
    - Counting the number of edge crossings with a ray starting at the queried point **P** till infinity
    - Inside, if the number of crossings is odd
  - (Non-zero) winding number rule
    - Counts # times polygon wraps around **P**
      - Signed intersections with a ray
    - Inside, if the number is not equal to zero
  - Differences only in the case of non-simple curves (e.g. self-intersection)



Winding



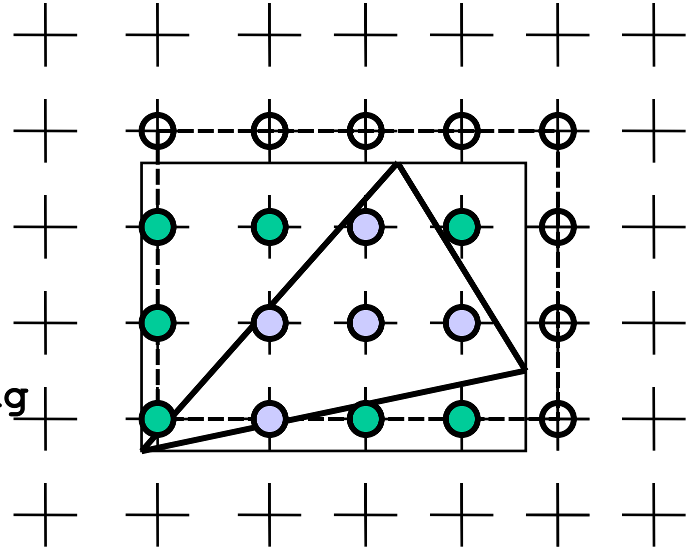
Even-odd



NZ-Winding

Triangle Rasterization

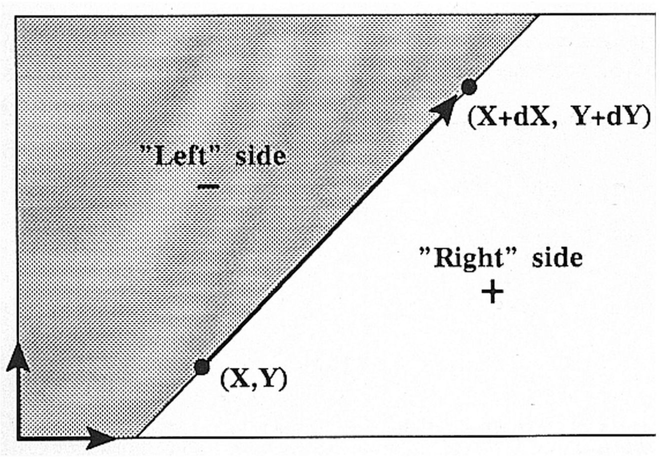
```
Raster3_box(vertex v[3])
{
    int x, y;
    bbox b;
    bound3(v, &b);
    for (y = b.ymin; y < b.ymax; y++)
        for (x = b.xmin; x < b.xmax; x++)
            if (inside(v, x, y)) // upcoming
                fragment(x,y);

}
```



- **Brute-force algorithm**
  - Iterate over all pixels within bounding box
- **Possible approaches for dealing with scissoring**
  - Scissoring: Only draw on AA-Box of the screen (region of interest)
    - Test triangle for overlap with scissor box, otherwise discard
    - Use intersection of scissor and bounding box, otherwise as above
    - Important if clipping only against enlarged region! (→ see later)

Rasterization w/ Edge Functions

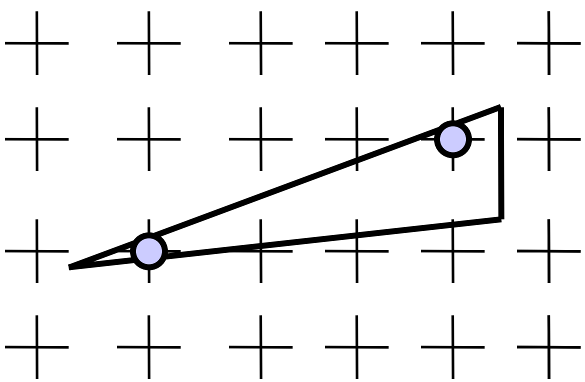
- **Approach (Pineda, `88)**
  - Implicit edge functions for every edge $F_i(x, y) = ax + by + c$
  - Point is *inside* triangle, if every $F_i(x, y)$ has the same sign
  - Perfect for parallel evaluation at many points
    - Particularly with wide SIMD machines (GPUs, SIMD CPU instructions)
  - Requires "triangle setup": Computation of 3 edge functions (*a, b, c*)
  - Evaluation can also be done in homogeneous coordinates



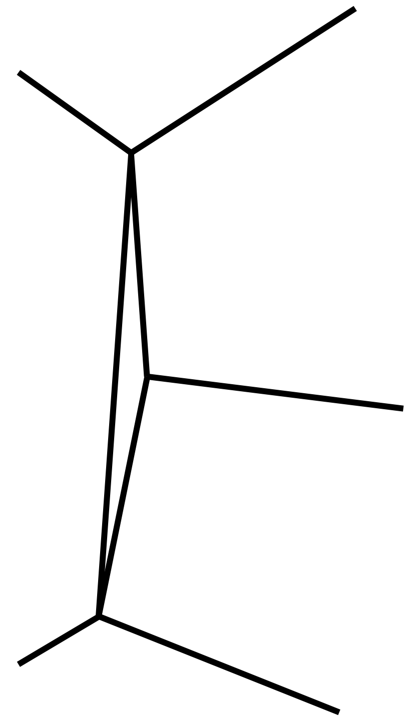
- **Hierarchical approach**
  - Can be used to efficiently check large rectangular blocks of pixels
    - Divide screen into tiles/bins (possibly at several levels)
    - Evaluate *F* at tile corners (making sure triangle is not completely inside)
    - Recurse only where necessary, possibly until subpixel level

Gap and T-Vertices

- **Observations**
  - Pixels set can be non-connected
  - May have overlap and gaps at T-edges



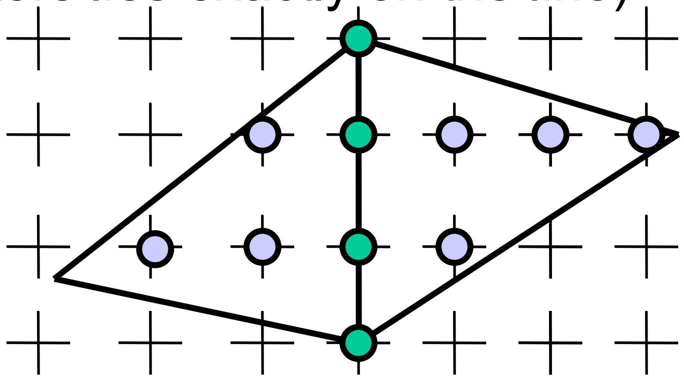
Non-connected pixels: OK



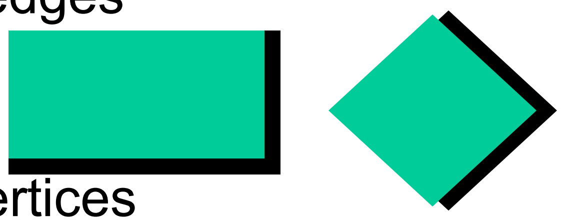
Not OK: Triangles must be changed

Problem on Edges

- **Consistency: edge singularity (shared by 2 triangles)**
  - What if term $d = ax+by+c = 0$ (pixel centers lies exactly on the line)
  - For $d <= 0$: pixels would get set twice
    - Problem with some algorithms
    - Transparency, XOR, CSG, ...
  - Missing pixels for $d < 0$ (set by no tri.)



- **Solution: “shadow” test**
  - Pixels are not drawn on the right and bottom edges
  - Pixels are drawn on the left and upper edges
    - Evaluated via derivatives *a* and *b*
  - Testing for all edges also solves problem at vertices



```
inside(value d, value a, value b)
{       // ax + by + c = 0
        return (d < 0) || (d == 0 && !shadow(a, b));
}
shadow(value a, value b)
{
        return (a > 0) || (a == 0 && b > 0);
}
```

Ray Tracing vs. Rasterization

- **In-Triangle test (for common origin)**
  - Rasterization:
    - Project to 2D, clip
    - Set up 2D edge functions, evaluate for each sample (using 2D point)
  - Ray tracing:
    - Set up 3D edge functions, evaluate for each sample (using direction)
  - The ray tracing test can also be used for rasterization in 3D
    - Avoids projection & clipping
- **Enumerating scene primitives**
  - Rasterization (simple):
    - Sequentially enumerate them all in any order
  - Rasterization (advanced):
    - Build (coarse) spatial index (typically on application side)
    - Traverse with view frustum (large)
      - Possibly one frustum for every image tile separately, when using *tiled rendering*
  - Ray Tracing:
    - Build (detailed) spatial index
    - Traverse with (infinitely thin) ray or with some (typically small) frustum
  - Both approaches can benefit greatly from spatial index!

Ray Tracing vs. Rasterization (II)

- **Binning (finding relevant pixels in a large frustum)**
  - Test to (hierarchically) find pixels likely covered by a primitive
  - Rasterization:
    - Great speedup due to very large view frustum (many pixels)
  - Ray tracing (frustum tracing)
    - Can speed up, depending on frustum size [Benthin'09]
  - Ray Tracing (single/few rays)
    - Not needed
- **Conclusion**
  - Both algorithms can use the same in-triangle test
    - In 3D, requires floating point, but boils down to 2D computation
  - Both algorithms can benefit from spatial index
    - Benefit depends on relative cost of in-triangle test (HW vs. SW)
  - Both algorithms can benefit from 2D binning to find relevant samples
    - Benefit depends on ratio of covered/uncovered samples per frustum
- **Both approaches are very similar**
  - Different organization (size of frustum, binning)
  - There is no reason RT needs to be slower for primary rays (exc. FP)

HW-Supported Ray Tracing (finally)

Imagination-Grafikchip: 5 Mal schneller als GeForce GTX 980 Ti beim Raytracing

heise online 11.01.2016 17:25 Uhr – Martin Fischer vorlesen

Fünf Mal schneller als eine GeForce GTX 980 Ti soll die Mobil-GPU PowerVR GR6500 sein, allerdings nur bei bestimmten Raytracing-Anwendungen.

Die Mobil-Grafikeinheit PowerVR GR6500 soll fünf Mal schneller arbeiten als Nvidias GeForce GTX 980 Ti bei nur einem Zehntel der Leistungsaufnahme; allerdings nur bei bestimmten Raytracing-Anwendungen.

HW-Supported Ray Tracing (finally)